Homework 3

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3. (a) Add -5 to both sides of $2x + 5 = 8$ and use (A2),(A4),(A3) to get $2x = 3$. Then multiply both sides by $1/2$ to get $x = 3/2$..
(b) Write $x^2 - 2x = x(x-2) = 0$ and apply Theorem 2.1.3(b). Alternatively, note that $x = 0$ satisfies the equation, and if $x \neq 0$, then multiplication by $1/x$ gives $x = 2$.

4. Clearly $a = 0$ satisfies $a \cdot a = a$. If $a \neq 0$ and $a \cdot a = a$, then $(a \cdot a)(1/a) = a(1/a)$, so that $a = a(a(1/a)) = a(1/a) = 1$.

8. (a) Let $x = m/n, y = p/q$, where $m, n \neq 0, p, q \neq 0$ are integers. Then $x + y = (mq + np)/nq$ and $xy = mp/nq$ are rational.
(b) If $s := x + y \in \mathbb{Q}$, then $y = s - x \in \mathbb{Q}$, a contradiction. If $t := xy \in \mathbb{Q}$ and $x \neq 0$, then $y = t/x \in \mathbb{Q}$, a contradiction.

10. (a) If $c = d$, then 2.1.7(b) implies $a + c < b + d$. If $c < d$, then $a + c < b + c < b + d$.
(b) If $c = d = 0$, then $ac = bd = 0$. If $c > 0$, then $0 < ac$ by the Trichotomy Property and $ac < bc$ follows from 2.1.7(c). If also $c \leq d$, then $ac \leq ad < bd$. Thus $0 \leq ac \leq bd$ holds in all cases.

19. The inequality is equivalent to $0 \leq a^2 - 2ab + b^2 = (a - b)^2$.

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6. (a) $|4x - 5| \leq 13 \iff -13 \leq 4x - 5 \leq 13 \iff -8 \leq 4x \leq 18 \iff -2 \leq x \leq 9/2$.
(b) $|x^2 - 1| \leq 3 \iff -3 \leq x^2 - 1 \leq 3 \iff -2 \leq x^2 \leq 4 \iff 0 \leq x^2 \leq 4 \iff -2 \leq x \leq 2$.

15. Choose any $\varepsilon > 0$ such that $\varepsilon < |a - b|$.