

Homework 2

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20. (a) If $f(x_1) = f(x_2)$, then $g(f(x_1)) = g(f(x_2))$, which implies $x_1 = x_2$, since $g \circ f$ is injective. Thus f is injective.

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3. It is true for $n = 1$ since $3 = 4 - 1$. If the equality holds for $n = k$, then add $8(k + 1) - 5 = 8k + 3$ to both sides and show that $(4k^2 - k) + (8k + 3) = 4(k + 1)^2 - (k + 1)$ to deduce equality for the case $n = k + 1$.
14. If $n = 4$, then $2^4 = 16 < 24 = 4!$. If $2^k < k!$ and if $k \geq 4$, then $2^{k+1} = 2 \cdot 2^k < 2 \cdot k! < (k + 1) \cdot k! = (k + 1)!$. [Note that the inductive step is valid whenever $2 < k + 1$, including $k = 2, 3$, even though the statement is false for these values.]