

1. If  $A \subseteq B$ , then  $x \in A$  implies  $x \in B$ , whence  $x \in A \cap B$ , so that  $A \subseteq A \cap B \subseteq A$ . Thus, if  $A \subseteq B$ , then  $A = A \cap B$ .

Conversely, if  $A = A \cap B$ , then  $x \in A$  implies  $x \in A \cap B$ , whence  $x \in B$ . Thus if  $A = A \cap B$ , then  $A \subseteq B$ .

8. (a)  $f(E) = \{1/x^2 : 1 \leq x \leq 2\} = \{y : \frac{1}{4} \leq y \leq 1\} = [\frac{1}{4}, 1]$ .  
(b)  $f^{-1}(G) = \{x : 1 \leq 1/x^2 \leq 4\} = \{x : \frac{1}{4} \leq x^2 \leq 1\} = [-1, -\frac{1}{2}] \cup [\frac{1}{2}, 1]$ .
11.  $E \setminus F = \{x : -1 \leq x < 0\}$ ,  $f(E) \setminus f(F)$  is empty, and  $f(E \setminus F) = \{y : 0 < y \leq 1\}$ .
13. If  $x \in f^{-1}(G) \cap f^{-1}(H)$ , then  $x \in f^{-1}(G)$  and  $x \in f^{-1}(H)$ , so that  $f(x) \in G$  and  $f(x) \in H$ . Then  $f(x) \in G \cap H$ , and hence  $x \in f^{-1}(G \cap H)$ . This shows that  $f^{-1}(G) \cap f^{-1}(H) \subseteq f^{-1}(G \cap H)$ . The opposite inclusion is shown in Example 1.1.8(b). The proof for unions is similar.