1. If the condition in 7.2.2(b) is satisfied, we can take $\eta = 1/n$ and obtain the condition in the statement. Conversely, if the statement holds and $\eta > 0$ is given, we can take $n \in \mathbb{N}$ such that $1/n < \eta$ to get the desired $\mathcal{P}_n, \mathcal{Q}_n$.

3. Let $\mathcal{P}_n$ be the partition of $[0, 1]$ into $n$ equal subintervals with $t_1 = 1/n$ and $\mathcal{Q}_n$ be the same subintervals tagged by irrational points.

5. If $c_1, \cdots, c_n$ are the distinct values taken by $\varphi$, then $\varphi^{-1}(c_j)$ is the union of a finite collection $\{J_{j_1}, \cdots, J_{j_{\tau_j}}\}$ of disjoint subintervals of $[a, b]$. We can write $\varphi = \sum_{j=1}^{n} \sum_{k=1}^{\tau_j} c_j \varphi J_{j_{k}}$.

8. If $f(c) > 0$ for some $c \in (a, b)$, there exists $\delta > 0$ such that $f(x) > \frac{1}{2} f(c)$ for $|x - c| \leq \delta$. Then $\int_a^b f \geq \int_{c-\delta}^{c+\delta} f \geq \frac{1}{2} f(c)(2\delta) > 0$. If $c$ is an endpoint, a similar argument applies.

11. Since $\alpha_c(x) = f(x)$ for $x \in [c, b]$, then $\alpha_c \in \mathcal{R}[c, b]$; similarly $\omega_c \in \mathcal{R}[c, b]$. The Additivity Theorem 7.2.8 implies that $\alpha_c$ and $\omega_c$ are in $\mathcal{R}[a, b]$. Moreover, $\int_a^b (\omega_c - \alpha_c) = 2M(c-a) < \varepsilon$ when $c-a < \varepsilon/2M$. The Squeeze Theorem 7.2.3 implies that $f \in \mathcal{R}[a, b]$. Further, $|\int_a^b f - \int_c^b f| = |\int_a^c f| \leq M(c-a)$.

13. Let $f(x) := 1/x$ for $x \in (0, 1]$ and $f(0) := 0$. Then $f \in \mathcal{R}[c, 1]$ for every $c \in (0, 1)$, but $f \notin \mathcal{R}[0, 1]$ since $f$ is not bounded.

15. Suppose $E = \{a = c_0 < c_1 < \cdots < c_m = b\}$. Since $f$ is continuous on the interval $(c_{i-1}, c_i)$, a two-sided version of Exercise 11 implies that its restriction is in $\mathcal{R}[c_{i-1}, c_i]$. The preceding exercise implies that $f \in \mathcal{R}[a, b]$. The case where $a$ or $b$ is not in $E$ is similar.

22. Let $x_i := i(\pi/2)$ for $i = 0, 1, \cdots, n$, so that $x_{n-i} = \pi/2 - x_i$ and $\sin x_{n-i} = \cos x_i$. Thus $(\pi/2n) \sum_{i=0}^{n-1} f(\cos x_i) = (\pi/2n) \sum_{k=1}^{n} f(\sin x_k)$. 
