4. Note that \( y_1 = 1 < \sqrt{3} = y_2 \), and if \( y_{n+1} - y_n > 0 \), then \( y_{n+2} - y_{n+1} = (y_{n+1} - y_n) / (\sqrt{2} + y_{n+1} + \sqrt{2} + y_n) > 0 \), so \((y_n)\) is increasing by Induction. Also \( y_1 < 2 \) and if \( y_n < 2 \), then \( y_{n+1} = \sqrt{2 + y_n} < \sqrt{2 + 2} = 2 \), so \((y_n)\) is bounded above. Therefore \((y_n)\) converges to a number \( y \) which must satisfy \( y = \sqrt{2 + y} \), whence \( y = 2 \).

6. Show that the sequence is monotone. The positive root of the equation \( z^2 - z - a = 0 \) is \( z^* := \frac{1}{2}(1 + \sqrt{1 + 4a}) \). Show that if \( 0 < z_1 < z^* \), then \( z_1^2 - z_1 - a < 0 \) and the sequence increases to \( z^* \). If \( z^* < z_1 \), then the sequence decreases to \( z^* \).

13. (a) \( (1 + 1/n)^n(1 + 1/n) \to e \cdot 1 = e \), (b) \([ (1 + 1/n)^n ]^2 \to e^2 \),
(c) \([ 1 + 1/(n + 1) ]^{n+1} / [ 1 + 1/(n + 1) ] \to e/1 = e \),
(d) \( (1 - 1/n)^n = [ 1 + 1/(n - 1) ]^{-n} \to e^{-1} = 1/e \).