1. Any negative number or 0 is a lower bound. For any \( x \geq 0 \), the larger number \( x + 1 \) is in \( S_1 \), so that \( x \) is not an upper bound of \( S_1 \). Since \( 0 \leq x \) for all \( x \in S_1 \), then \( u = 0 \) is a lower bound of \( S_1 \). If \( v > 0 \), then \( v \) is not a lower bound of \( S_1 \) because \( v/2 \in S_1 \) and \( v/2 < v \). Therefore \( \inf S_1 = 0 \).

2. \( S_2 \) has lower bounds, so that \( \inf S_2 \) exists. The argument used for \( S_1 \) also shows that \( \inf S_2 = 0 \), but that \( \inf S_2 \) does not belong to \( S_2 \). \( S_2 \) does not have upper bounds, so that \( \sup S_2 \) does not exist.

3. Since \( 1/n \leq 1 \) for all \( n \in \mathbb{N} \), then 1 is an upper bound for \( S_3 \). But 1 is a member of \( S_3 \), so that \( 1 = \sup S_3 \). (See Exercise 6 below.)

4. \( \sup S_4 = 2 \) and \( \inf S_4 = 1/2 \). (Note that both are members of \( S_4 \).)

5. If \( S \) is bounded below, then \( S' := \{-s : s \in S\} \) is bounded above, so that \( u := \sup S' \) exists. If \( v \leq s \) for all \( s \in S \), then \( -v \geq -s \) for all \( s \in S \), so that \( -v \geq u \), and hence \( v \leq -u \). Thus \( \inf S = -u \).

6. Let \( u \in S \) be an upper bound of \( S \). If \( v \) is another upper bound of \( S \), then \( u \leq v \). Hence \( u = \sup S \).

7. If \( t > u \) and \( t \in S \), then \( u \) is not an upper bound of \( S \).

8. Let \( u := \sup S \). Since \( u \) is an upper bound of \( S \), so is \( u + 1/n \) for all \( n \in \mathbb{N} \). Since \( u \) is the supremum of \( S \) and \( u - 1/n < u \), then there exists \( s_0 \in S \) with \( u - 1/n < s_0 \), whence \( u - 1/n \) is not an upper bound of \( S \).

9. Let \( u := \sup A, v := \sup B \) and \( w := \sup\{u, v\} \). Then \( w \) is an upper bound of \( A \cup B \), because if \( x \in A \), then \( x \leq u \leq w \), and if \( x \in B \), then \( x \leq v \leq w \). If \( z \) is any upper bound of \( A \cup B \), then \( z \) is an upper bound of \( A \) and of \( B \), so that \( u \leq z \) and \( v \leq z \). Hence \( w \leq z \). Therefore, \( w = \sup(A \cup B) \).

10. Since \( \sup S \) is an upper bound of \( S \), it is an upper bound of \( S_0 \), and hence \( \sup S_0 \leq \sup S \).