2.1

1. (a) Apply appropriate algebraic properties to get \( b = 0 + b = (-a + a) + b = -a + (a + b) = -a + 0 = -a. \)
   (b) Apply (a) to \((-a) + a = 0\) with \( b = a \) to conclude that \( a = -(−a) \).
   (c) Apply (a) to the equation \( a + (-1)a = a(1 + (-1)) = a \cdot 0 = 0 \) to conclude that \((-1)a = -a.\)
   (d) Apply (c) with \( a = -1 \) to get \((-1)(−1) = -(−1).\) Then apply (b) with \( a = 1 \) to get \((-1)(−1) = 1.\)

6. Note that if \( q \in \mathbb{Z} \) and if \( 3q^2 \) is even, then \( q^2 \) is even, so that \( q \) is even. Hence, if \( (p/q)^2 = 6 \), then it follows that \( p \) is even, say \( p = 2m \), whence \( 2m^2 = 3q^2 \), so that \( q \) is also even.

7. If \( p \in \mathbb{N} \), there are three possibilities: for some \( m \in \mathbb{N} \cup \{0\} \), (i) \( p = 3m \), (ii) \( p = 3m + 1 \), or (iii) \( p = 3m + 2 \). In either case (ii) or (iii), we have \( p^2 = 3h + 1 \) for some \( h \in \mathbb{N} \cup \{0\} \).

13. If \( a \neq 0 \), then 2.1.8(a) implies that \( a^2 > 0 \); since \( b^2 \geq 0 \), it follows that \( a^2 + b^2 > 0. \)

22. (a) Let \( x := c − 1 > 0 \) and apply Bernoulli's Inequality 2.1.13(c) to get \( c^n = (1 + x)^n \geq 1 + nx \geq 1 + x = c \) for all \( n \in \mathbb{N} \), and \( c^n > 1 + x = c \) for \( n > 1. \)
   (b) Let \( b := 1/c \) and use part (a).

25. Let \( b := c^{1/mn}. \) We claim that \( b > 1; \) for if \( b \leq 1 \), then Exercise 22(b) implies that \( 1 < c = b^{mn} \leq b \leq 1, \) a contradiction. Therefore Exercise 24(a) implies that \( c^{1/n} = b^m > b^n = c^{1/m} \) if and only if \( m > n. \)