Final Review - MA 401

Ch 1 - very slight
2 - Fourier Series (at least 1/3 of exam)
3 - Separation of Variables, Eigenfunctions
4 - Bessel's DE, Bessel functions, Series
5 - Legendre Polynomials (5.2, 5.5)
6 - 6.1 - orthogonal functions

Fourier Series -

change of variable, manipulating known Fourier series to get what you need.
half range expansions
uniform convergence, interchanging limit and integration (differiation)

$f(x) = a\left(1-\left(\frac{x}{p}\right)^2\right)$ on $[-p,p]$, $a \neq 0$

has Fourier series $\frac{2}{3}a + 4a\sum_1^\infty \frac{(-1)^{n+1}}{(n\pi)^2}\cos\frac{n\pi x}{p}$

Find Fourier series for $g(x) = x^2$ on $[-p,p]$

Let $a=1$: $1-\left(\frac{x}{p}\right)^2 = 1-\frac{x^2}{p^2} = \left(\frac{p^2-x^2}{p^2}\right)$

$$p^2 f(x) = p^2 - x^2$$

$$x^2 = p^2 - p^2 f(x)$$

$$= p^2 - p^2\left(\frac{2}{3} + 4\sum_1^\infty \frac{(-1)^{n+1}}{(n\pi)^2}\cos\frac{n\pi x}{p}\right)$$

$$= \frac{1}{3}p^2 - 4p^2\sum_1^\infty \frac{(-1)^{n+1}}{(n\pi)^2}\cos\frac{n\pi x}{p}$$

Uniform convergence

Show $f_n(x) = e^{-nx}\cos x$ converges uniformly on $[0,\infty)$

Find $\lim f_n$:

$x=0$: $f_n(0) = e^0 \cos 0 = 1 \to 1$

$x>0$: $|f_n(x)| = |e^{-nx}\cos x| \le e^{-nx} \to 0$

$\lim\limits_{n\to\infty} e^{-nx} = 0$

$$f(x) = \begin{cases} 1 & x = 0 \\ 0 & x > 0 \end{cases}$$

$f_n \to f$ uniformly on $[0,\infty)$ iff

$|f_n(x) - f(x)| \underset{n}{\to} 0$ for all $x \ge 0$. ①

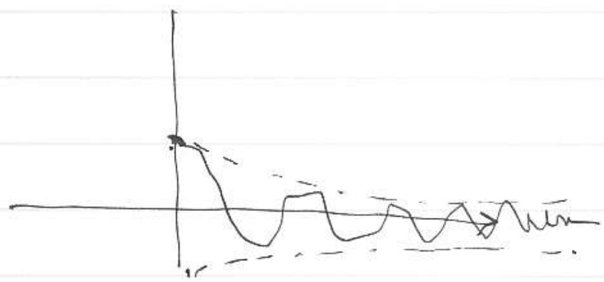
or Take maximum on $x \in [0,\infty)$ of ②

$|f_n(x) - f(x)|$, then take limit on n.

$$|f_n(x) - f(x)| = \begin{cases} |f_n(0) - f(0)| & \\ |f_n(x) - f(x)| & x > 0 \end{cases}$$

$= |f_n(x) - f(x)|$, $x > 0$.

Maximum $|e^{-nx}\cos nx| \le e^{-nx} = 1 = f_n(0)$



$\lim\limits_{n\to\infty} 1 = 1 \neq 0$. Hence $f_n \not\to f$ unif. on $[0,\infty)$

On $[1, \infty)$, $|f_n(x) - f(x)|$ has maximum

$$e^{-n \cdot 1} \cos 1 = e^{-n} \cos 1 \qquad (x=1)$$

$$\lim_{n \to \infty} e^{-n} \underbrace{\cos 1}_{\text{constant}} = 0 \qquad f_n \to f \text{ uniformly on } [1, \infty)$$

Spring systems
Parseval's Identity

Ch. 3 - Separation of variables
will have separation of variables problem

Fourier Series
double Fourier series
generalized Fourier series
Poisson's eqn — Eigenfunction expansion - Not a separation of variables problem.

Ch. 4 - Bessel functions, series

Ch. 5 - Legendre functions

$$(1-x^2)\, y'' - 2x y' + \mu y = 0 \qquad \text{Legendre DE}$$

Euler's DE $x^2 y'' + \alpha x y' + \beta y = 0$