IE/MA/OR 505 Linear Programming

Prerequisites: Elementary matrix algebra (at the level of MA 305 or MA 405), and elementary multivariable calculus (at the level of MA 242).

Content: An introduction, including applications to some problems that arise in engineering, operations research and management science, but with emphasis on basic solution techniques and fundamental theory. Topics include: (1) the simplex method and its main variants, as well as a very brief description of the more recently developed interior-point methods; (2) parametric programming and post-optimality analysis; (3) duality, including applications to matrix games and linear systems solvability; (4) polyhedral sets and cones, including their convexity and separation properties, as well as their dual representations; (5) Lagrange multipliers and equilibrium prices, as well as subgradients and sensitivity analysis.

Text: Lecture notes made available by the instructor, supplemented by approximately 23 reference books made available at the reserve desk of Hill Library.

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Instructor:  E. L. Peterson
  Office hours:  12:35 - 1:35 PM on Tuesdays and Thursdays,
    beginning in the classroom and continuing in
    Harrelson Hall 206 if necessary, or by appointment.
  Phone:  881-9194 (only when necessary).
  E-mail address:  elpeters@eos.ncsu.edu (only when a
    single-sentence response is feasible).

Teaching assistant:  Elina Bryksina
  Office hours:  2:00 - 3:00 PM on Mondays, Tuesdays
    and Thursdays in Daniels 451, or by appointment.
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    single-sentence response is feasible).
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Grades: Normally determined by two midterm exams and a final exam that counts twice. In terms of the average of the three highest of these four exam scores, the transition from A to B (through A-and B+) is an average of approximately 90%, from B to C 80%, and so forth. A relatively high final exam score and/or relatively strong performances on the homework (if graded) can result in a higher letter grade.

Course Outline

A. Optimization examples and general models
   1. Activity analysis and resource allocation
   2. Diet selection
   3. Scaffolding analysis
   4. Transportation and assignment
   5. Network optimization

B. Fundamental concepts and terminology
   1. Equivalent problems
      a. Slack variables
      b. Standard formulation
      c. Canonical problems
      d. Reduction to canonical form

C. Simplex algorithm (both the uncontracted form using Dantzig tableaus and the contracted form using Tucker schemas)
   1. Pivot algebra (using both tableaus and schemas)
      a. Basic solutions and basic feasible solutions
      b. Adjacent basic solutions and adjacent basic feasible solutions
   2. Determination of inconsistency or a feasible solution (phase I)
   3. Determination of unboundedness or an optimal solution (phase II)
   4. Finite termination via the prevention of circling (using tableaus)
      a. Lexicographic ordering and lexicographic minimization
   5. Pivot matrices (for tableaus)
   6. Tableau solution sequences
      a. Computational checks
   7. Revised simplex algorithm, including its product form
   8. Computational complexity, with comparisons to the computational complexity of the competing interior-point algorithms

D. Parametric programming and post-optimal sensitivity analysis

E. Duality
   1. Economic interpretations
   2. The dual problem and primal-dual schemas

3. The dual simplex algorithm
4. Computational considerations

F. Fundamental theorems
1. Existence of feasible solutions (both primal and dual)
2. Existence and uniqueness of optimal solutions (both primal and dual)
3. Duality gaps
4. Optimality conditions (via complementary slackness)

G. Two-person zero-sum matrix games
1. Pure strategies
2. Mixed strategies
3. Saddle solutions
4. Equivalent dual problems
5. Minimax description

H. Solvability theory for finite linear systems (as time permits)

I. Geometry of linear optimization (as time permits)
1. Hyperplanes and half-spaces
2. Polyhedral sets and polytopes
3. Affine sets, convex sets and cones
4. Affine hulls, convex hulls and conical hulls
5. Resolution theorems for convex polyhedral sets and polytopes

J. Convexity in linear optimization (as time permits)
1. Convex functions, including their subgradients and directional derivatives
2. Optimal-value subgradients, equilibrium prices, and Lagrange multipliers characterized as dual-optimal solutions
3. Post-optimal sensitivity analysis revisited

REFERENCES

Elementary Introductory References

1. Bertsimas, D. and Tsitsiklis, J.N., *Introduction to Linear Optimization*
   Belmont, Mass.: Athena Scientific, 1997
2. Chong, E.K.P and Zak, S.H., *An Introduction to Optimization*
   New York: Wiley, 1996.
3. Dantzig, G.B. and Thapa, M. N., *Linear Programming*
   Springer, New York, 1997.
4. Hillier, F.S and Lieberman, G.J., *Introduction to Operations Research*
   McGraw-Hill, New York, 1995.
5. Kolman, B. and Beck, R.E., *Elementary Linear Programming with Applications*
   Academic Press, San Diego, 1995.
6. Taha, H.A., *Operations Research: An Introduction*
   Prentice-Hall, Upper Saddle River, NJ, 1997.

Advanced Introductory References

1. Bazaraa, M.S. and Jarvis, J.J., *Linear Programming and Network Flows*
   John Wiley and Sons, New York, 1977
2. Chvatal, V., *Linear Programming*
   Freeman, New York, 1983.
3. Dantzig, G.B., *Linear Programming and Extensions*
   Princeton University Press, NJ, 1963.
4. Dorfman, R., Samuelson, P.A. and Solow, R.M., *Linear Programming and Economic Analysis*
   McGraw-Hill, New York, 1975.
5. Fang, S.C. and Puthenpura, S., *Linear optimization and Extensions*
   Prentice-Hall, 1993.
6. Gale, D., *Theory of Linear Economic Models*
   McGraw-Hill, New York, 1960.
7. Gass, S.I. , *An Illustrated Guide to Linear Programming*, Illus. By McWilliam, W.F.
   McGraw-Hill, New York, 1975.
8. Hadley, G., *Linear Programming*
   Addison-Wesley, Reading, Mass., 1962.
9. Luenberger, D.G., *Introduction to Linear and Non-Linear Programming*
   Addison-Wesley, Reading, Mass. 1973.

10. Murty, K., *Linear and Combinatorial Programming*
John Wiley and Sons, New York, 1976.
11. Salkin, H.M. and Saha, J., *Studies in Linear Programming*
North-Holland Publishing Company, Amsterdam, 1975.\
12. Simonnard, M., *Linear Programming*, English translation by Jewell, W.S.
Prentice-Hall, Eaglewood Cliffs, NJ, 1966.
13. Spivey, W.A. and Thrall, R.M., *Linear Optimization*
Holt, Rinehard and Winston, 1970.
14. Strum, J.E., *Introduction to Linear Programming*
Holden-Day, San Francisco, 1977.
15. Tucker, A.W. and Nering, E.D., *Linear Programs and related problems*
Boston :Academic Press, 1993.
16. Van de Panne, C., *Linear Programming and Related Techniques*
North-Holland Publishing Company, Amsterdam, 1976.
17. Zukhovitskiy, S.I. and Avdeeva, L.I., *Linear and Convex Programming*
W.B. Saunders Company, 1966.