On the Limitation of Fluid-based Approach for Internet Congestion Control

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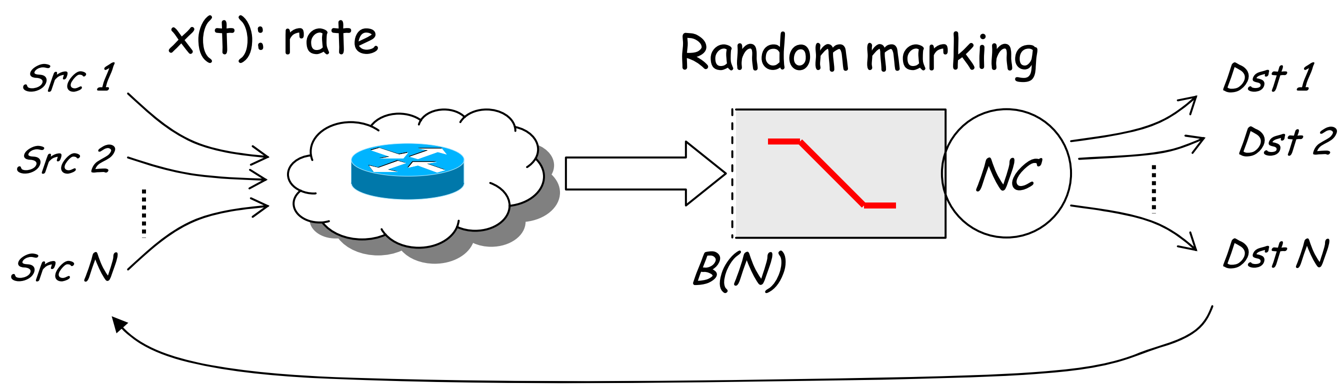
Outline

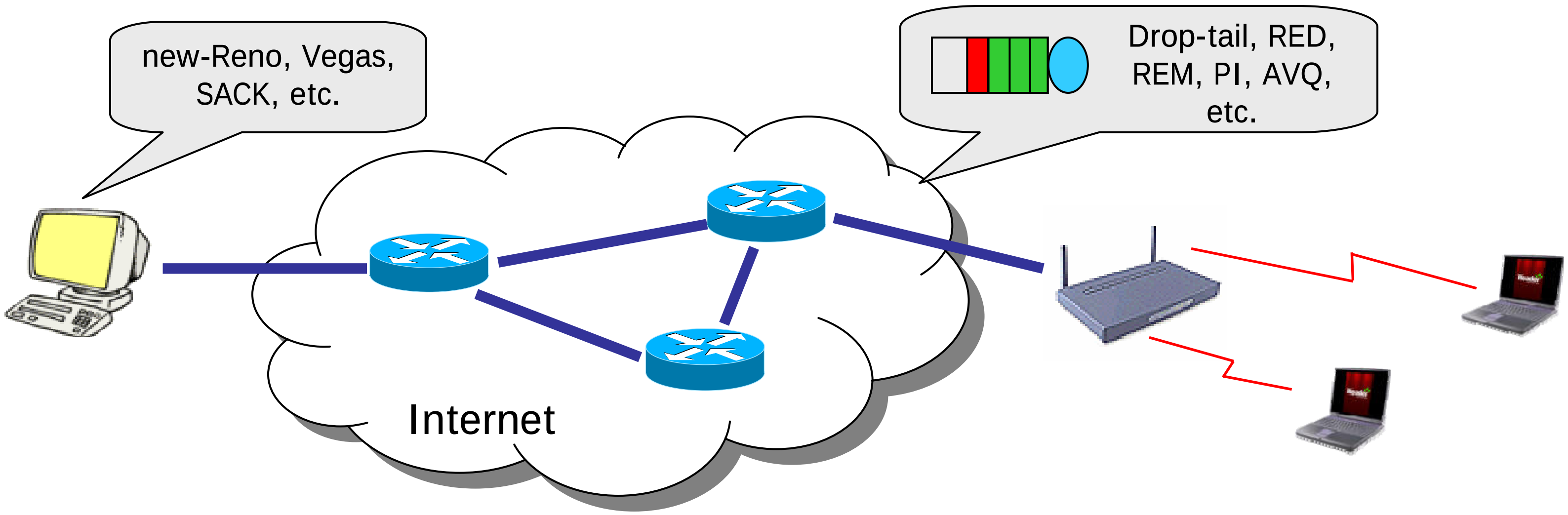
- TCP/AQM Congestion Control
- Fluid vs. Stochastic Approach
  - Equilibrium Points
  - Stability Implications
- Discrepancy between Two Approaches
- Stochastic Approach Works Better
- Summary & Conclusion

TCP/AQM Congestion Control



- More than 90% traffic carried via TCP
- It's a feedback system (equilibrium, stability)
- Two approach for analysis and design of TCP/AQM:
  - Fluid-based Approach
  - Stochastic Approach

TCP/AQM Primer



TCP:

- Slow start: probe exponentially for bandwidth
- Congestion avoidance:
  - Send **w** packets in a round-trip time.
  - If no congestion, then put **w+1** packets in next RTT
  - If congestion, put **w/2** packets in next RTT.

AQM:

- Governs how to generate the congestion signal based on input rate or queue-length
- ECN marks

Fluid Approach for Congestion Control

$$\frac{dx(t)}{dt} = \kappa[w - x(t)p(x(t))]$$

$\Leftrightarrow$

$$\max_{x_i \geq 0} \sum_i U_i(x_i)$$
$$\text{s.t.} \sum_{i \in l} x_i \leq c_l$$

- Very popular in networking literature
- Provides analysis tools and design guidelines
  - Congestion control, wireless networks, cross-layer design, etc.
- TCP/AQM: distributed solution to utility maximization problem (with some notion of fairness)
- Equilibrium point (fixed point), Stability (convergence)

Fluid Approach for Congestion Control

$$\frac{dx(t)}{dt} = \kappa[\underbrace{w}_{\text{AI}} - \overbrace{x(t)p(x(t))}^{\text{MD}}]$$

Canonical form of AIMD

- Deterministic differential/difference equations
- Equilibrium point → predicts target operating points
- Stability (global or local) → provides design guidelines
- Captures "Average quantities" (x(t): window size or throughput, etc)

Fluid Model in Discrete Time

Single flow case:

average rate or window size

$$x(t+1) = \underbrace{(x(t)}_{}+1)(1-p(x(t))) + \frac{x(t)}{2}p(x(t))$$

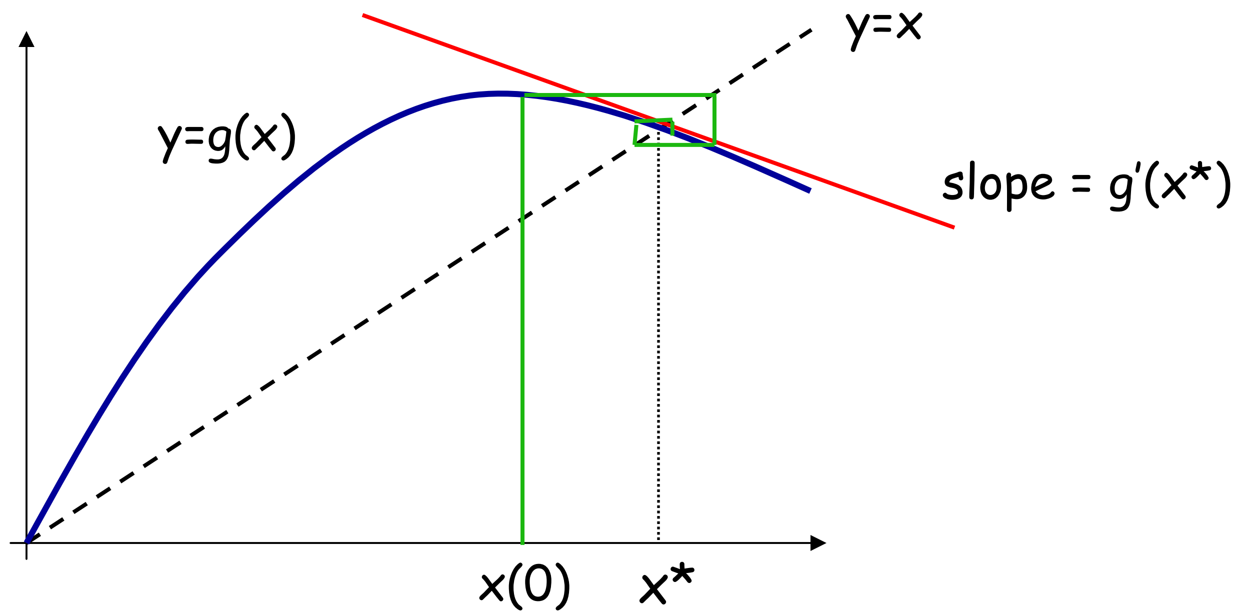
$$:= g(x(t))$$

prob. of congestion

where $g(x) := (x+1)(1-p(x)) + xp(x)/2$

- p(x): probability of receiving marks or loss (congestion signal)
  - $p(x) = (x/C)^B \wedge 1$ → P{Q>B} for M/M/1 (Poisson arrival)
  - $p(x) = \exp[-2B(C-x)/\sigma^2 x] \wedge 1$ → Gaussian arrival with mean x and var. $\sigma^2 x$
- Given current "rate" x, p(x) models random packet arrivals

Equilibrium and Stability of Fluid Model



- Equilibrium point (fixed point) x^*: $x^* = g(x^*)$
- Linear stability: $x(t+1) = g(x(t))$ converges locally if and only if $|g'(x^*)| < 1$ (locally `contractive')

Markovian Model

$$X(t+1) = \begin{cases} X(t)+1, & \text{w.p. } 1-p(X(t)) \\ X(t)/2, & \text{w.p. } p(X(t)) \end{cases}$$

Canonical form of AIMD

- Markovian description: Given current state, the next state is obtained probabilistically
- Rate (window size) is always +1 or /2, nothing in between
- X(t) will never converge! But its distribution does.
- Stability → Ergodic Markov process
- Equilibrium → stationary distribution π of X(t)

Equivalent Representation

- Let $f(x,u) := (x+1)(1-1_{\{u \le p(x)\}}) + \frac{x}{2} 1_{\{u \le p(x)\}}$
- Then,

$$X(t+1) = f(X(t), U_t) \quad t = 1, 2, \ldots$$

Markov Process

U_t: i.i.d. unif [0,1]

- Fluid model x(t+1) = g(x(t)) becomes

$$\begin{aligned} x(t+1) &= E\Big\{X(t+1) \;\Big|\; X(t) = x(t)\Big\} \\ &= \int_0^1 f(x(t), u)\ du = g(x(t)) \end{aligned}$$

Deterministic value

  - Fluid model x(t) captures "average" of X(t)

Equilibrium and Stability of Markov Model

- The previous Markov process always converges in total variation to π
  - Guaranteed by Foster's criterion
  - π : stationary distribution of X(t); very hard to find…
  - Starting from any initial distribution, X(t) converges to a steady-state in which X(t) has a stationary distribution π
- Let $\widehat{x}$ be the average rate in the steady-state, i.e.,

$$\widehat{x} = \mathbb{E}_{\pi}\{X(t)\}$$

- If X(t) is bounded (as usual), then

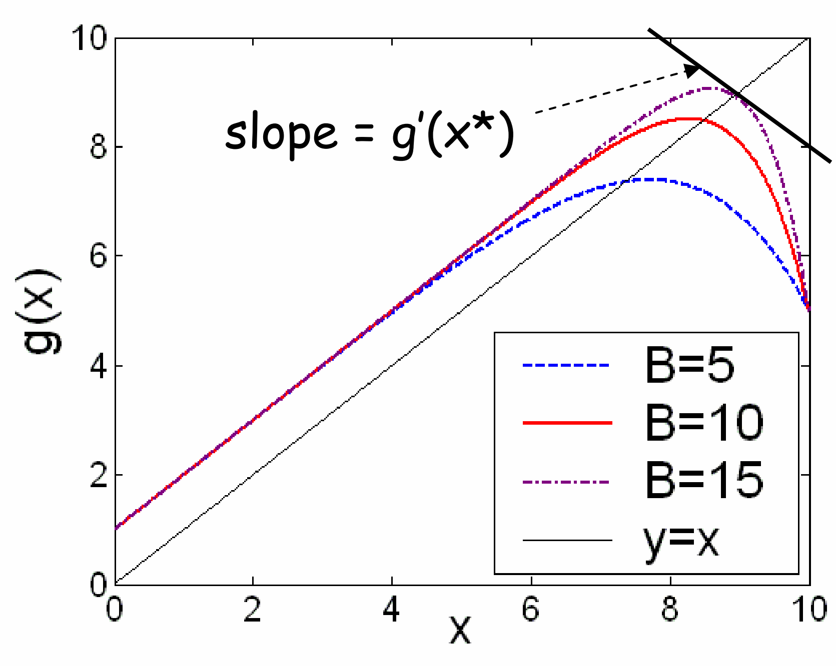
$$\mathbb{E}\{X(t)\} \longrightarrow \widehat{x} \quad \text{as} \quad t \to \infty$$

Discrepancy in Equilibrium Points

- The fluid model captures "average" of X(t)
- We expect that $x(t) \approx E\{X(t)\}$
- Suppose the fluid model is stable, i.e., $x(t) \to x^*$
- If the fluid model were to be "close" to the original Markov model in capturing the "average" behavior, we should expect $x^* = \widehat{x}$, i.e., both approaches predicts the same equilibrium point.
- **Proposition 1:** If g(x) is either strictly convex or concave, we have

$$x^* \neq \widehat{x}$$

Examples: Discrepancy in Equilibrium



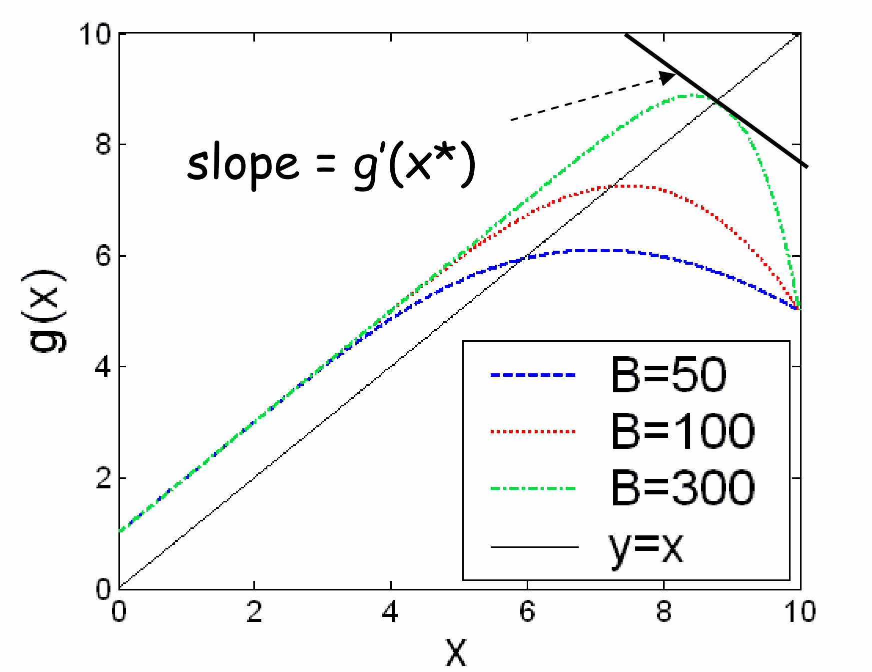
- M/M/1 type arrivals

$$p(x) = \left(\frac{x}{C}\right)^B$$

- C = 10
- Fluid model is locally stable for B=5,10,15

	x*	$\hat{x} = \mathbb{E}_\pi\{X(t)\}$	$(x^* - \hat{x})/x^*$
B=5	7.34	6.30	14.1%
B=10	8.47	7.11	16%
B=15	8.93	7.35	17.7%

Examples: Discrepancy in Equilibrium



- Brownian type arrivals

$$p(x) = \exp\left(\frac{-2B(C-x)}{\sigma^2 x}\right)$$

- $C = 10$, $\sigma^2 = 50$
- Fluid model is locally stable for B=50,100,300

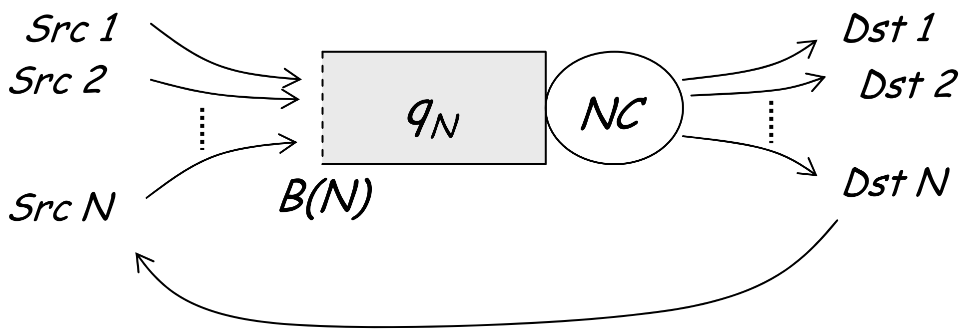
	x*	$\hat{x} = \mathbb{E}_\pi\{X(t)\}$	$(x^* - \hat{x})/x^*$
B=50	5.92	5.28	11%
B=100	7.23	6.29	13%
B=300	8.76	7.28	17%

Discrepancy in Equilibrium Points

For single flow case (Summary):

- Fluid approximation of original Markov process yields (i) equilibrium point x^* and (ii) stability condition
- In general, $x^* \neq E_\pi\{X(t)\}$
- Even for "stable" fluid models with $x(t) \rightarrow x^*$, the x^* is different from the "true average" value.
  - Then, what is x^* ?

Fluid Model for Multiple Flows



- $x_i(t)$: rate (window size) of flow i (i=1,2, …, N)
- $p(\cdot)$ depends only on the average rate (over N)

$$x_i(t+1) = (x_i(t)+1)\left(1 - p\left(\underbrace{\frac{\sum_{i=1}^N x_i(t)}{N}}\right)\right) + \frac{x_i(t)}{2} p\left(\frac{\sum_{i=1}^N x_i(t)}{N}\right)$$

= $y_N(t)$: Average rate on the link

for each i

$$\Longrightarrow \quad y_N(t) = (y_N(t)+1)(1-p(y_N(t))) + \frac{y_N(t)}{2} p(y_N(t))$$

Same as the single flow case!

Stability of Rate-Based AQM

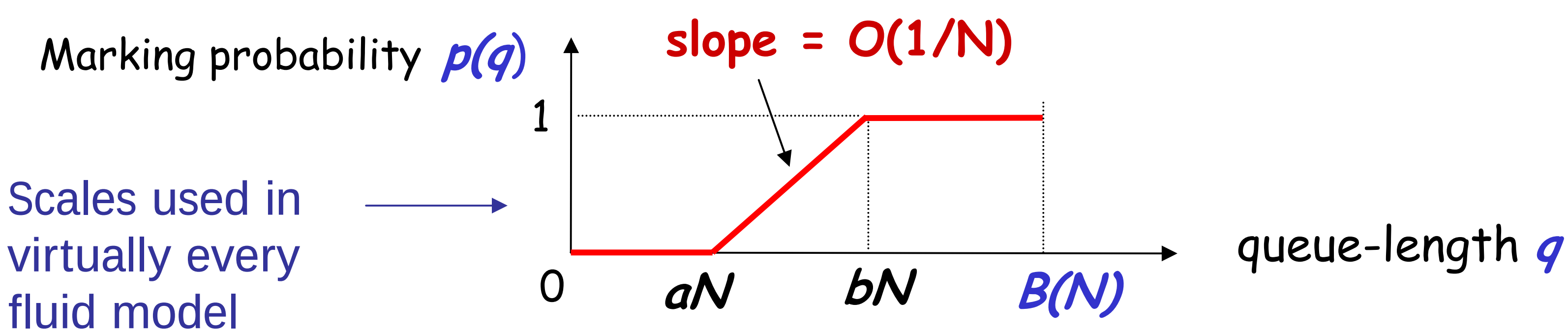
- Average rate $y_N(t)$ satisfies the same fluid equation as the single flow case
  - Same equilibrium point $y^*_N = x^*$ (regardless of N!)
  - Same stability condition
- Stability condition:
  - $p(x) = (x/C)^B$ → Fixed point $x^* < C$
  - Stability condition: $B < B'$ (for some constant B')
  - Bounded buffer size for stability, at the cost of reduced link utilization $\rho < 1$
  - Similar observations (Kunniyur, Srikant, Deb)

Stability of Queue-Based AQM

- The function p(·) depends on $q_N(t)/N$, where

$$q_N(t+1) = \left[q_N(t) + N y_N(t) - NC\right]^+$$

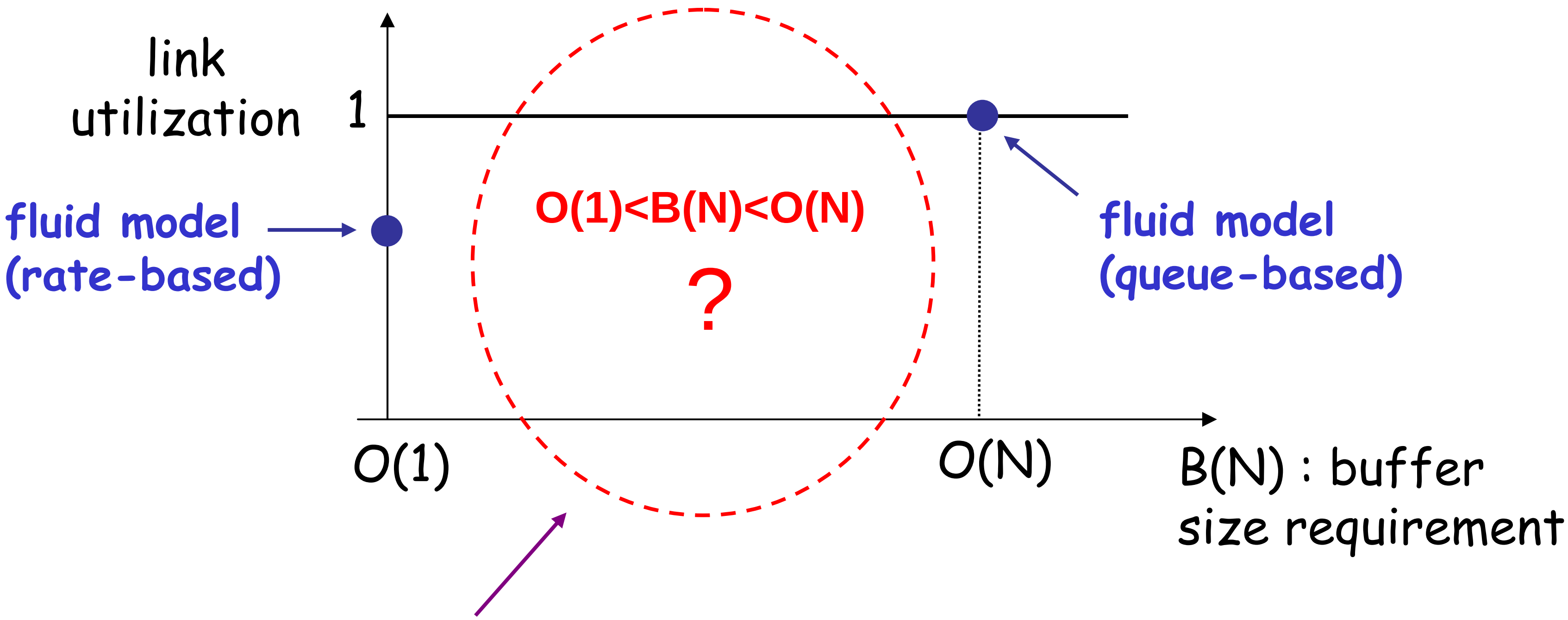
- This means that, for stability, slope of the marking function p(·) at the fixed point should be O(1/N) [Low, Srikant, Shakkottai]



- Buffer size then should be at least B(N) = O(N)
  - rule-of-thumb: B(N) = NC × RTT = O(N)

Trade-off: Buffer Size vs. Utilization

- Trade-off between buffer size and link utilization for <u>(linearly) stable fluid models</u>



Linearly "unstable" for both models ➡ prohibited ?

Markovian Model for Many Flows

$$X_i(t+1) = \begin{cases} (X_i(t)+1) \wedge w_{max}, & \text{w.p. } 1 - p(Y_N(t)) \\ (X_i(t)/2) \vee 1, & \text{w.p. } p(Y_N(t)), \end{cases}$$

where $Y_N(t) := \frac{1}{N}\sum_{i=1}^{N} X_i(t)$

- N-dimensional Markov process: $(X_1(t), X_2(t), \ldots, X_N(t))$
- For any given N, the above chain is ergodic.
- Since $Y_N(t) \leq w_{max}$, we expect that

$$\lim_{t\to\infty} \mathbb{E}\{Y_N(t)\} = \mathbb{E}_\pi\{Y_N\} = \hat{y}_N$$

Regardless of initial distribution of $Y_N(t)$

Behavior in the Steady-State

- In the steady-state, under weak-dependency among X_i, we have $\sum_{i=1}^{N} X_i = N\hat{y}_N + o(N)$

error term

$$\Longrightarrow \quad Y_N = \hat{y}_N + o(1)$$

Law of large numbers

- Similarly, the average marking probability will also converge, i.e., $\mathbb{E}\{p(Y_N(t))\} \longrightarrow p_N$
- We expect that $0 < \alpha \leq p_N \leq \beta < 1$
  - Constants $\alpha, \beta \in (0,1)$
  - If $p_N \approx 0$, then almost all flows increase rates in the next RTT $\rightarrow$ distribution will not be stationary.
  - Similar argument for the case of $p_N \approx 1$

Behavior in the Steady-State

- Take $p(y) = (y/C)^{B(N)}$
  - Poisson packet arrivals to queue with capacity NC and buffer size B(N)
- Previous expression gives

$$\left(\frac{\hat{y}_N}{C} + o(1)\right)^{B(N)} = \kappa(N), \quad \text{where} \quad 0 < \alpha \leq \kappa(N) \leq \beta < 1$$

- So,

avg. rate for MC

error factor

$$\hat{y}_N = C \cdot \left(\kappa(N)^{\frac{1}{B(N)}} - o(1)\right)$$

target avg. rate for full utilization

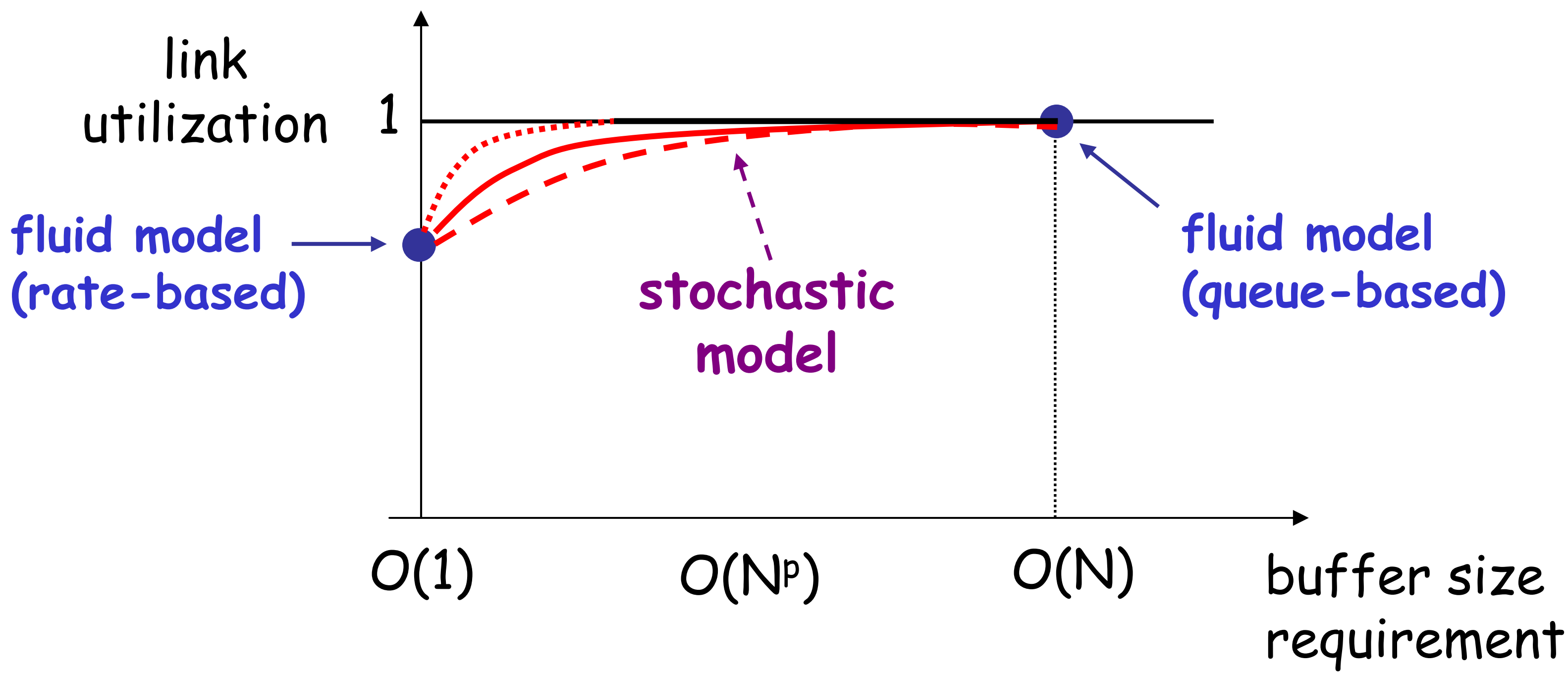
Trade-off: Buffer Size vs. Utilization

$$\hat{y}_N = C \cdot \left(\kappa(N)^{\frac{1}{B(N)}} - o(1) \right)$$

- $\kappa(N) > 0$ is bounded away from 0
- $\hat{y}_N \rightarrow C$ as long as B(N)$\rightarrow \infty$
- Achieve full link utilization for any increasing function B(N) for the buffer size
- System is always "stochastically stable"
- **No such trade-off as in the fluid model!**

Fluid vs. Stochastic Models

- Buffer size vs. Link utilization tradeoff for a `stable' system with N flows and capacity NC



Some Evidence from the Literature

- [Appenzeller, et. al. 04]
  - Under drop-tail, high link utilization under

$$B(N) = O(\sqrt{N})$$

  - Empirically observed independence among flows
  - Has nothing to do with stability of any kind…
- [Eun & Wang 05]
  - Under various queue-based AQMs, high link utilization and low packet loss under

$$B(N) = O(N^{\alpha}) \qquad \text{where} \quad 0 < \alpha < 0.5$$

  - Based on stochastic models and stochastic stability (ergodicity)

Summary & Conclusion

- Fluid approach is versatile and powerful
- Actual behavior in the network is more like "stochastic".
- Fluid approach may be limited and result in
  - Inaccurate equilibrium
  - Excessive restriction of system parameters
  - Tradeoff between utilization and buffer size
- No such tradeoff in stochastic approach
  - Results in much wider system parameter choices with good performance
  - Evidence: Recent results on buffer sizing, etc.

Thank You!

Questions ?