Crossing Over the Bounded Domain: From Exponential To Power-law Inter-meeting time in MANET

Han Cai, Do Young Eun
Department of Electrical and Computer Engineering
North Carolina State University
Motivation – inter-meeting time

Significance of Inter-meeting time

- One of contact metrics (especially important for DTN)
Motivation – exp. inter-meeting

- Assumed for tractable analysis [1, 2]
- Supported by numerical simulations based on mobility model (RWP) [3, 4]
- Theoretical result to upper bound first and second moment [5] using BM model on a sphere


Motivation – power-law inter-meeting (1)

- Recently discovered: power-law [6, 7]

Effect of power-law on system performance [6]

“If $\alpha < 1$, none of these algorithms, including flooding, can achieve a transmission delay with a finite expectation.”

Effect of infrastructure and multi-hop transmission [8]

“... A consequence of this is that there is a need for good and efficient forwarding algorithms that are able to make use of these communication opportunities effectively.”

Recent study on power-law (selected)

- Call for new mobility model [6]
  - Use 1-D random walk model to produce power-law inter-meeting time [9]

- Call for new forwarding algorithm [8]

Our work

- What’s the fundamental reason for exponential & power-law behavior?

- In this paper, we
  - Identify what causes the observed exponential and power-law behavior
  - Mathematically prove that most current synthetic mobility models necessarily lead to exponential tail of the inter-meeting time distribution
  - Suggest a way to observe power-law inter-meeting time
  - Illustrate the practical meaning of the theoretical results
Content

- Inter-meeting time with exponential tail
- From exponential to power-law inter-meeting time
- Scaling the size of the space
- Simulation
The inter-meeting time $T_I$ of nodes $A$ and $B$ is defined as:

$$T_I \triangleq \inf_{t>0} \{ t : \|A(t) - B(t)\| \leq d \}$$

given that $\|A(0) - B(0)\| = d$ and $\|A(0^+) - B(0^+)\| > d$.

Two nodes under study are independent, unless otherwise specified.
Random Waypoint Model

- We consider
  - Zero pause time
  - Random pause time (light-tail)
**Proposition 1**: Under zero pause time, there exists constant $c > 0$ such that

$$P\{T_I > t\} < e^{-ct},$$

for all sufficiently large $t$.

- Proposition 1 is also true for “bounded” pause time case.
Proof sketch for Proposition 1

Independent “Image” (snapshot of node positions)

- \( W_1 = W_2 = \cdots = \zeta \)
- \( \# \text{ of independent “image”} = O(t) \)
- Each “image”: \( P \{ \text{not meeting} \} < c < 1 \)
Random pause time: the difficulty

Independent “Image”

\[ Z_1 = Z_2 = \ldots = \zeta \]

\[ \text{# of independent “image”} = O(t) \]
**Theorem 1:** Under random pause time, there exists constant \( c > 0 \) such that

\[
P\{T_I > t\} < e^{-ct},
\]

for all sufficiently large \( t \).

- Proposition 1 is extended to random pause time case, i.e., the pause time may be infinite.
Random Walk Models (MC)

- Markov Chain RWM: transition matrix
  \[ P = \{ p_{ij} \}, \]
  prob. of jumping from cell i to j

- Boundary behavior
  - Reflect
  - Wrap around

- Two node meet if and only if they are in the same cell

General version of discrete isotropic RWM
Assumptions on RWM

- After deleting any single state from the MC model, the resulting state space is still a communicating class.
  - The failure of any one cell will not disconnect the mobility area - if an obstacle is present, the moving object (people, bus, etc.) will simply bypass it, rather than stuck on it.

- For any possible trajectory of node B, node A eventually meets node B with positive probability (No conspiracy).
**Theorem 2:** Suppose that node A moves according to the RWM and satisfies assumptions on RWM. Then, there exists constant $\gamma > 0$ such that

$$P\{T_I > t\} \leq e^{-\gamma t},$$

for all sufficiently large $t$.

- Only one node is required to move as RWM.
  - Theorem 2 applies to inter-meeting time of two nodes moving as: RWM+RWM, RWM+RWP, RWM+RD, RWM+BM, etc.
- Effect of spatial constraints (e.g., obstacles) is also reflected (by assigning $p_{ij}$).
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Common factor leads to exponential tail?

What is common in all these models?
Common factor leads to exponential tail?

Finite Boundary!!!

- “Boundary” is incorporated in definition
  - RWM: wrapping or reflecting boundary behavior
  - RWP: boundary concept inherited in model definition (destination for each jump is uniformly chosen from a bounded area)
Finite boundary: exponential tail

- Two nodes not meet for a long time
  → most likely move towards different directions
  → prolonged inter-meeting time
  <strong>memory</strong>

- Finite boundary erase this memory <strong>memoryless</strong>
Other factors than boundary?

- For most current synthetic models, finite boundary critically affects tail behavior of inter-meeting time.

- Other possible factors:
  - Dependency between mobile nodes
  - Heavy-tailed pause time (with infinite mean)
  - Correlation in the trajectory of mobile nodes

- Our study focuses on:
  - Independence case
  - Weak-dependence case
Removing the boundary …

- **Isotropic random walk in $\mathbb{R}^2$**
  - Choose a random direction uniformly from $[0, 2\pi)$
  - Travel for a random length in $(0, \infty)$
  - Repeat the above process

**Theorem 4**: Two independent nodes $A, B$ move according to the 2-D isotropic random walk model described above. Then, there exists constant $C > 0$ such that the inter-meeting time $T_I$ satisfies:

$$P\{T_I > t\} \geq Ct^{-1/2},$$

for all sufficiently large $t$. 
Proof sketch for Theorem 4 (1)

- **1-D isotropic random walk**
  - $P \{\text{jump left over } L\} = P \{\text{jump right over } L\}$
  - First passage time: starting from a non-origin $x_0$, minimum time to return to the origin

- **Sparre-Andersen Theorem**: For any one-dimensional discrete time random walk process starting at non-origin $x_0$ with each step chosen from a continuous, symmetric but otherwise arbitrary distribution, the First Passage Time Density (FPTD) to the origin asymptotically decays as $t^{-1.5}$. 
Proof sketch for Theorem 4 (2)

- Difference walk
- Find lower bound
  - $T_F \leq T_I$
- Map to 1-D
  - $C(t) \rightarrow [C(t)]_x$
- Apply S-A Theorem
Content

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- Scaling the size of the space

- Simulation
Questions

- About the boundary
  - In reality, all domain under study is bounded
  - In what sense does “infinite domain” exist?

- About exponential/power-law behavior
  - Where does the transition from exponential to power-law happen?
The interaction between the timescale under discussion and the size of the boundary

- Position of node A (following 2-D isotropic random walk) at time \( t \): \( A(t) \), satisfies 
  \[ \mathbb{E}\{\|A(t)\|^2\} = t \]

- “Average amount of displacement”: standard deviation of \( A(t) \), scales as \( O(\sqrt{t}) \)

- Standard BM: position scale as \( O(\sqrt{t}) \)
BM: time/space scaling

- Area: 800X800 m²

- Is 200X200 domain bounded?
  - Unbounded over time scale [0,100]
  - Bounded over time scale [0,1000000]

- KEY: whether the boundary effectively "erases" the memory of node movement
Content

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- Simulation
RWM: $P\{T_I > t\}$ (log-log)

- RWM: change direction uniformly every 50 seconds
- Speed: $U(1.00, 1.68)$
- Simulation period $T$: 40 hours
- Avg. amount of displacement: 500m
RWM: $P\{T_I > t\}$ (linear-log)

Essentially unbounded domain
\[ \Rightarrow \text{Power-law behavior} \]

Essentially bounded domain
\[ \Rightarrow \text{Exponential behavior} \]
Irrespective of the domain size, the tail of inter-meeting exhibits an exponential behavior.

For either zero pause or random pause cases, the slope of the CCDF decreases as domain size increases.
“Finite boundary” is a decisive factor for the tail behavior of inter-meeting time, we prove

- The exponential tailed inter-meeting time based on RWP, RWM model
- The power-law tailed inter-meeting time after removing the boundary

Time/space scaling, i.e., the interaction between domain size and time scale under discussion is the key to understand the effect of boundary
Thank You!

Questions?