

A Regime Switching Analysis of the Exchange Rate Pass-through

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Abstract

We investigate the stability of the pricing policies of exporters. This includes the stability in the exchange rate pass-through coefficient as well as the stability in the response to other variables that affect the firm's markup. The model assumes that in every period exporters set prices by following either a "high pass-through" or a "low pass-through" pricing policy. The transition from one policy to the other is governed by a Markov process; thus it implies "stickiness" to the policy. Relying on the theoretical literature on determinants of the optimal choice of exchange rate pass-through, we assume that the transition probabilities of the Markov process depend on economic fundamentals. We estimate the model using collected data on 35 lines of imported cars to the US from seven exporting countries for the 1980-2004 period. Our estimations suggest that the "low pass-through" regime is characterized by: a low exchange rate pass-through; a low response to misalignments in the firm's relative price; a low volatility of technology and preference shocks; and a higher duration—of about two years and three quarters—than the other regime. We identify the significant factors behind the choice of a pricing policy as US inflation, country market share, and market concentration. We also find that US inflation and the volatility of the exchange rate are important factors in explaining the decline in the average exchange rate pass-through in the automobile industry.

1 Introduction

There is a growing body of empirical literature that documents the decline in the exchange rate pass-through rates in various industries and for a number of countries (Campa and Goldberg (2005), Frankel, Parsley and Wei (2005), Marazzi and Sheets (2007)). While the important implications of the decline in the pass-through for adjustments of external imbalances and monetary policy are acknowledged, the sources of the decline are not fully established. Using industry level data from OECD countries, Campa and Goldberg (2005) have shown that the changes in the composition of exports can provide some explanation to this decline. On the other hand, the findings by Marazzi and Sheets (2007), emphasize the structural changes in export pricing behavior of firms brought about by global competition in explaining the decline. Changes in the decisions of exporters about how much of the exchange rate fluctuations to absorb and how much to transmit in their prices would affect the pass-through both at the disaggregated and industry level.

Theoretical studies suggest a number of economic fundamentals that determine an exporting firm's optimal degree of exchange rate pass-through.¹ While some of the studies focus on product or industry specific characteristics, others highlight the role of macroeconomic conditions. Some of the product and/or industry specific factors that have been studied are substitutability of the export good (e.g., Giovannini (1988), Donnenfeld and Zilcha (1991), Friberg (1998), Goldberg and Tille (2005)), strategic complementarities and the market share of the exporting country (e.g., Feenstra, Gagnon and Knetter (1996), Bacchetta and van Wincoop (2005)). The macroeconomic indicators that are important for the degree of pass-through have been identified as monetary stability and inflation (Taylor (2000), Devereux, Engel and Storgaard (2004)). As one or more of these factors vary across time for different exporters, the pricing policies of exports can change, which in turn would affect the degree of exchange rate pass-through.

¹The two theoretical polar cases of exchange rate pass-through are producer currency pricing (PCP) and local currency pricing (LCP). If all exporting firms choose set their prices in their home currencies (PCP), then they will be passing on all the fluctuations in the exchange rates onto the consumers; therefore, the pass-through will be complete. If, on the other hand, all the firms choose the importing country's currency (LCP), then there will be no pass-through.

The aim of this paper is to empirically investigate the factors that affect the degree of exchange rate pass-through at a disaggregated level, using a panel dataset of U.S. automobile imports. The challenge in disentangling different factors behind the degree of exchange rate pass-through is that the pricing policies of exporters are unobservable. One needs to build inferences not only about the optimal pricing decisions, but also about the underlying economic fundamentals, which can affect the pricing decisions non-linearly. We employ the hidden regime switching methodology popularized by Hamilton (1990, 1994) and its later extension by Diebold, Lee and Weinbach (1994) to build inferences about the underlying fundamentals along with the pricing decisions.

We start by theoretically formulating the optimal export pricing decision of a firm under exchange rate uncertainty. One important implication that comes out of our formulation is that, keeping everything else constant, the degree of exchange rate pass-through chosen by the firm affects other variables in the pricing policy. Allowing for strategic complementarities, the choice degree of exchange rate pass-through affects the marginal cost pass-through in addition to the sensitivity to changes in the competitors prices. Hence, instead of focusing only on the potential changes of the pass-through parameter, we investigate the stability of the export pricing policies of the foreign automobile manufacturers.

We follow Diebold, Lee and Weinbach's (1994) methodology in estimating the parameters in the pricing policy. We assume that in each period the exporting firm is subject to a random shock that follows a two-state, first-order Markov process with time varying transition probabilities. These transition probabilities between the two states are determined by exogenous variables that are suggested by the theories of pricing decisions (product differentiation, inflation, exchange rate volatility, market share, etc.). The random shock triggers one of the two different sets of parameter values in the optimal pricing policy, and hence determines the pricing regime. We refer to the two pricing policies resulting from those sets of parameters as the "high pass-through" pricing regime and the "low pass-through" pricing regime. The actual state of the firm is unobservable: we only observe the actual price, but do not observe the pricing regime that it comes from. Nevertheless, we can estimate the probability

of being in one regime versus the other along with the pricing equations. The estimated probabilities show which factors are significant in determining the degree of pass-through. Furthermore, analyzing the trends in those factors, we investigate the role of each of the factors in the decline of the exchange rate pass-through.

Our dataset consists of 35 automobile models from 7 exporting countries for the 1980-2004 period. For each automobile we observe the manufacturer's suggested retail price, sales, physical characteristics and location of production. Having detailed information on individual products allows us to control for the homogeneity of the good across time. The manufacturer's suggested retail price is net of any retail or transportation cost. Therefore, the exchange rate pass-through coefficients we estimate are the at-the-dock rates.²

Our estimation method identifies two different pricing regimes that are characterized by: (i) significantly different exchange rate pass-through coefficients; (ii) lower (higher) sensitivity to misalignments in the firm's relative price in the low (high) pass-through regime; (iii) lower (higher) volatility of technology and preference shocks faced by firms in the low (high) pass-through regime. These findings are robust to including different variables in the transition probabilities.

Initially, we allow the transition probabilities to be a function of a constant and a single factor. The inflation rate in the US, exporting country's market share and market concentration are estimated to be highly significant determinants of the transition probabilities. While there is weak evidence for the importance of each automobile line's segment share (as an indicator of the substitutability of the good), the inflation rate in the exporter's country does not seem to be a significant factor behind pricing policies. Using the conditional probabilities we estimate from each of these specifications, we build inferences about the fraction of firms in the low pass-through regime for each year in our sample. Fluctuations in the exchange rate volatility and US inflation predict a systematic increase in the fraction of firms in low pass-through through-out the sample. The evidence is mixed for other determinants.

²The percentage of exchange rate fluctuations that the exporters choose to transmit is interpreted as the pass-through "at the dock". The exchange rate pass-through to the final consumption goods depend also on local costs. See, for example, Burnstein, Neves and Rebelo (2003) and Corsetti and Dedola (2004).

In a more general specification that allows the transition probabilities to be functions of inflation, country share and market concentration, and we analyze the dynamic behavior of the average pass-through in the automobile industry. We find a non-monotonic, downward trend in the exchange rate pass-through with the highest average pass-through coefficient of 13.4% in 1987, and the lowest average pass-through coefficient of 6.4% in 2001.

The paper is organized as follows: next section develops the theoretical export pricing equation we use in our estimations. Section 3 presents the empirical methodology we use. Section 4 describes the sources of our data and talks about the variables that we construct. Results are presented and interpreted in Section 5. Finally, conclusions are presented in Section 6.

2 Pricing of Exports

The aim of this section is to develop an optimal pricing equation for our empirical framework and to highlight the notion that the changes in the degree of exchange rate pass-through can also imply changes in the other parameters. Below we build a simple theoretical based pricing equation that shows the instability of the parameters, and motivates our choice of regime switching for empirical work.

Assume that there is a continuum of monopolistically competitive firms selling to the US market indexed by $\ell \in \mathbb{L}_t$ formed by foreign and domestic firms.³ At time $t - 1$ a representative foreign firm optimally chooses the price for the product that will be sold at time t , $p_{\ell t}^*$, denominated in producer's own currency; then in period t the price is converted to US dollars using the following transformation:

$$p_{\ell t} = (e_t)^\delta p_{\ell t}^*, \tag{1}$$

where $p_{\ell t}$ is the price in US dollars that the consumer faces in the US market, e_t is the nominal exchange rate and the parameter $\delta \in [0, 1]$ measures the degree of exchange rate pass-through in the

³Note that in general, in the spirit of Feenstra (1994) we allow for changes in the mass (i.e. the number) of varieties in the market, but in particular we focus on the pricing problem of a single foreign firm in the US market.

export prices. Note that $\delta = 0$ implies that the firm is setting the export price in US dollars, hence is using local currency pricing (LCP); and $\delta = 1$ implies that it is using producer currency pricing (PCP). Thus, our pricing specification contains as special cases two pricing schemes often used in the literature: LCP and PCP.

Assume that the demand for the good ℓ produced by the firm ℓ is a function of its relative price $p_{\ell t}/P_{\mathbb{L}t}$, where $P_{\mathbb{L}t}$ is the price index of the industry, as well as a function of a vector of variables $\underline{Y}_{\ell t}$ such as consumer's income or consumer's preference shocks—thus not all elements in the vector $\underline{Y}_{\ell t}$ are firm specific. Let

$$Q\left(\frac{p_{\ell t}}{P_{\mathbb{L}t}}; \underline{Y}_{\ell t}\right) \quad (2)$$

be the demand function and let $\eta\left(\frac{p_{\ell t}}{P_{\mathbb{L}t}}; \underline{Y}_{\ell t}\right) \equiv -\frac{\partial Q(\cdot)}{\partial p_{\ell t}} \frac{p_{\ell t}}{Q(\cdot)} > 0$ be the price elasticity of demand. The production technology exhibits constant returns to scale. In absence of technology shocks, the real marginal cost is $\frac{\psi_{\ell t}^*}{P_t^*}$, where $\psi_{\ell t}^*$ is the nominal marginal cost denominated in foreign currency and P_t^* is the price index of the foreign consumption basket; however we allow for exogenous technology shocks $Z_{\ell t}^*$ that may disturb production costs; thus the effective real marginal cost is $\frac{\psi_{\ell t}^*}{P_t^*} \frac{1}{Z_{\ell t}^*}$. The producer chooses $p_{\ell t}^*$ to maximize expected real profits subject to the transformation (1) and the demand function (2). Thus the producer maximizes:

$$\mathbb{E}_{t-1} \frac{\Upsilon_t}{P_t^*} \left\{ \frac{p_{\ell t}}{e_t} Q\left(\frac{p_{\ell t}}{P_{\mathbb{L}t}}; \underline{Y}_{\ell t}\right) - \frac{\psi_{\ell t}^*}{Z_{\ell t}^*} Q\left(\frac{p_{\ell t}}{P_{\mathbb{L}t}}; \underline{Y}_{\ell t}\right) \right\},$$

where Υ_t is the relevant stochastic discount factor between $t - 1$ and t .

To obtain a log-linear approximation of the model let $\hat{x}_t \equiv \frac{dx_t}{\bar{x}}$ be deviations of the variable x_t from its steady-state \bar{x} . Thus, a log-linear approximation of the first-order condition, around the steady-state,

yields⁴

$$\hat{\Phi}_{\ell t} \equiv -\frac{1}{\bar{\eta} - 1} \hat{\eta}_{\ell t},$$

and $\bar{\eta} \equiv \eta(\cdot)|_{ss} > 1$ is the price elasticity evaluated at steady-state values. Thus, the exporter's optimal price denominated in dollars can be decomposed in three components. The first term in (??) captures the exchange rate surprise that the producer passes to its price, i.e. δ captures the degree of exchange rate pass-through. The second term, is the expected marginal cost in terms of dollars. The third term, $\hat{\Phi}_{\ell t}$, captures the demand-side effects that may affect the gap between the firm's price and its marginal cost in dollar terms; thus, we refer to $\hat{\Phi}_{\ell t}$ as the firm's markup. In turn, the markup is a decreasing function of the price elasticity.

Different functional forms for the utility function will imply different determinants for the markup term. Three notable utility specifications often used in the related literature are the Dixit-Stiglitz aggregator, the Dotsey-King aggregator and the translog utility index. The utility index proposed in Dixit and Stiglitz (1977) delivers a CES demand with constant markup; thus $\hat{\Phi}_{\ell t} = 0 \forall t$.⁵ The latter two utility specifications deliver time varying mark-ups. Both under the Dotsey and King (2005) aggregator (which builds on Kimball (1995)), and the translog utility index proposed in Feenstra (1994) the price elasticity of demand is a function of the firm's price and the price of competitors. Hence, the optimal price in those settings respond to the marginal cost, the exchange rate and the price of competitors summarized by the price index of the industry.⁶

Note that in our general specification, the markup is a function of the firm's relative price $\frac{p_{\ell t}}{P_{\ell t}}$ and other determinants of demand summarized in the vector $\underline{Y}_{\ell t}$; thus the optimal price also accounts for

⁴The first-order condition implies $\mathbb{E}_{t-1} \left\{ \frac{\Upsilon_t}{P_t^*} (e_t)^{\delta-1} Q(\cdot) [\eta(\cdot) - 1] \right\} = \mathbb{E}_{t-1} \left\{ \frac{\Upsilon_t}{P_t^*} \frac{\psi_{\ell t}^*}{Z_{\ell t}^*} \eta(\cdot) Q(\cdot) / p_{\ell t}^* \right\}$; moreover, in absence of uncertainty we obtain the well known condition $p_{\ell t} = \frac{\eta(\cdot)}{\eta(\cdot)-1} \psi_{\ell t}^* e_t$.

⁵Note that this feature does not depend on our assumption of one-period-ahead preset prices, for example, assuming Calvo pricing and a CES demand we can show that the optimal price, up to a first-order approximation, only depends on the current and expected marginal costs. Actually, the Dixit-Stiglitz aggregator is widely adopted in macroeconomic models of the business cycle precisely for its tractability.

⁶Devereux Engel and Storgaard (2004) use a Dixit-Stiglitz aggregator in a model of the endogenous choice of exchange rate pass-through. Gust, Leduc and Vigfusson (2006) use a Dotsey-King aggregator in a model that shows that trade integration produces a decline in the exchange rate pass-through. Using a translog utility index Feenstra (1996) shows that there is a non-linear relation between pass-through and the market share—market share of source country.

the expected impact in the firm's markup due to changes in the firm's relative price. We make this explicit by using a log-linear approximation of the markup together with equation (??) to obtain a first-order approximation of the optimal price: ⁷

$$\hat{p}_{\ell t} = \delta \hat{s}_t + \frac{1}{1 + \kappa(\delta)} \mathbb{E}_{t-1} \{ \hat{\psi}_{\ell t} - \hat{Z}_{\ell t}^* \} + \frac{\kappa(\delta)}{1 + \kappa(\delta)} \mathbb{E}_{t-1} \{ \hat{P}_{L t} \} + \frac{\chi(\delta)}{1 + \kappa(\delta)} \mathbb{E}_{t-1} \{ \hat{Y}_{\ell t} \} \quad (3)$$

where

$$\hat{s}_t = \hat{e}_t - \mathbb{E}_{t-1} \hat{e}_t$$

is the exchange rate surprise, and

$$\hat{\psi}_{\ell t} \equiv \hat{\psi}_{\ell t}^* + \hat{e}_t,$$

is the exchange-rate-adjusted nominal marginal cost. $\kappa(\delta) \equiv \frac{\bar{\eta}_1}{\bar{\eta}(\bar{\eta}-1)} \frac{e^{\delta} \bar{p}^*(z)}{P^L}$ makes explicit that the coefficient is a function of the exchange rate pass-through; $\bar{\eta}_1 \equiv \frac{\partial \eta(\cdot)}{\partial (p_t(z)/P_t^L)} \Big|_{ss}$; $\chi(\delta) \equiv -\frac{\partial \eta(\cdot)}{\partial Y_{\ell t}} \Big|_{ss} \frac{\bar{X}}{\bar{\eta}(\bar{\eta}-1)}$ is a column vector of parameter values; recall that $\hat{Y}_{\ell t}$ is a column vector of variables; and the upper bar indicates that the functions are evaluated at steady-state values.

In order to have a positive marginal cost pass-through coefficient we must constraint $\kappa(\delta) > -1$. Moreover, for prices to be strategic complements in the sense that the optimal price increases when the average price of competitors increases⁸, as in Bergin and Feenstra (1999), we further require $\kappa(\delta) > 0$. In turn, $\kappa(\delta) > 0$ requires $\bar{\eta}_1 > 0$, that is, as in Dotsey and King (2005) or Gust, Leduc and Vigfusson (2006), in steady-state, the price elasticity must be increasing in the firm's relative price.

Equation (3) makes transparent that the instability of the exchange rate pass-through parameter can bring instability to other parameters of the price equation.⁹ The optimal pricing implies that the

⁷Note that $\hat{\Phi}_{\ell t} = -\kappa \{ \hat{p}_{\ell t}^* + \delta \hat{e}_t - \hat{P}_{L t} \} + \hat{Y}'_{\ell t} \chi$, where $\chi \equiv -\frac{\partial \eta(\cdot)}{\partial Y_{\ell t}} \Big|_{ss} \frac{\bar{X}}{\bar{\eta}(\bar{\eta}-1)}$ and $\frac{\partial \eta(\cdot)}{\partial Y_{\ell t}}$ is a column vector of derivatives of the price-elasticity with respect to the vector $Y_{\ell t}$.

⁸See Bratsiotis (2007) and references therein.

⁹Note that in the special case of a CES demand $\kappa(\delta) = \chi(\delta) = 0$, thus in that special case the instability of the exchange rate pass-through can be studied in isolation, as in Devereux et al. (2004).

degree of the exchange rate pass-through also can affect the marginal cost pass-through and the sensitivity of the firm’s mark-up to the average price of the competitors. Thus, this theoretical framework points towards an empirical setting general enough to jointly analyze the stability of the exchange rate pass-through coefficient together with the stability of other parameters given the decision rules for the choice of degree of pass-through. The regime switching model presented in the following section has such characteristics.

3 A Regime Switching Model of Price Setting and its Estimation

3.1 Empirical Framework

We use the optimal pricing policy (3) as our benchmark to build an empirical framework. We assume that in each period t the foreign firm ℓ is subject to a random shock $\xi_{\ell t} \in \{0, 1\}$ that follows a two-state first-order Markov process. The random shock $\xi_{\ell t}$ triggers one of the two different sets of parameter values in the optimal pricing policy (3): $\{\delta_{\xi_{\ell t}}, \kappa(\delta_{\xi_{\ell t}}), \chi(\delta_{\xi_{\ell t}})\}$, and hence determines the pricing regime. We refer to the two pricing policies resulting from those sets of parameters as the “high pass-through” pricing regime and the “low pass-through” pricing regime. The actual state of the firm ℓ is unobservable to the econometrician, she only observes the actual price but does not observe from which pricing regime it comes from. It is worth to mentioning that our assumption of a first-order Markov process for $\xi_{\ell t}$, implies history dependence in the adoption of a particular pricing policy. Thus estimates of the transition probability matrix of the Markov process provide estimates of the degree of persistence or stickiness of the high (low) pass-through pricing regime.

Accordingly, based on (3), we model observed changes in the optimal price denominated in dollars

of the variety ℓ , $\Delta\hat{p}_{\ell t} \equiv \hat{p}_{\ell t} - \hat{p}_{\ell t-1}$, as:

$$\Delta\hat{p}_{\ell t} = \beta_{\xi_t}^s \Delta\hat{s}_t + \beta_{\xi_t}^\psi \Delta\hat{\psi}_{\ell t} + \beta_{\xi_t}^{P_L} \Delta\hat{P}_{L,t} + \Delta\hat{Y}'_{\ell t} \beta_{\xi_t}^y + \varepsilon_{\xi_{\ell t}} \quad (4)$$

where $\Delta\hat{s}_t$ is a proxy for the change in the exchange rate surprise; $\Delta\hat{\psi}_{\ell t}$ is a proxy for the expected change in the exchange-rate-adjusted nominal marginal cost; $\Delta\hat{P}_{L,t}$ is a proxy for the expected change in the average price of the competitors; and $\Delta\hat{Y}_{\ell t}$ is a column vector of proxies for expected changes in other variables that may affect the firm's markup.¹⁰ Section 4 describes in detail how we construct the variables utilized in our estimations.

The error term $\varepsilon_{\xi_{\ell t}}$ follows a standard normal distribution, $\varepsilon_t \sim i.i.d.\mathfrak{F}(0, \sigma_{\xi_{\ell t}}^2)$. Note that the error term $\varepsilon_{\xi_{\ell t}}$ contains unobservable technology shocks, $\hat{Z}_{\ell t}^*$ in equation (3), as well as unobservable demand shocks or preference shocks for which we cannot control in the vector $\Delta\hat{Y}_{\ell t}$, thus we can only identify the variance $\sigma_{\xi_{\ell t}}^2$ as the variance of an aggregate of both, technology and preference shocks.

Following Diebold, Lee and Weinbach (1994), we assume that the transition probability matrix that governs the two-state Markov process $\xi_{\ell t} \in \{0, 1\}$ contains the following elements:

$$g_{\ell t}^{00} \equiv \Pr(\xi_{\ell t} = 0 | \xi_{\ell t-1} = 0) = \mathfrak{B}(\underline{z}'_{\ell t-1} \phi_0),$$

$$g_{\ell t}^{11} \equiv \Pr(\xi_{\ell t} = 1 | \xi_{\ell t-1} = 1) = \mathfrak{B}(\underline{z}'_{\ell t-1} \phi_1), \quad (5)$$

$$g_{\ell t}^{01} \equiv \Pr(\xi_{\ell t} = 0 | \xi_{\ell t-1} = 1) = 1 - g_{\ell t}^{11},$$

$$g_{\ell t}^{10} \equiv \Pr(\xi_{\ell t} = 1 | \xi_{\ell t-1} = 0) = 1 - g_{\ell t}^{00} \quad (6)$$

where $\mathfrak{B}(x) = \frac{\exp(x)}{1+\exp(x)}$ is the logistic function, $\underline{z}_{\ell t}$ is a vector of economic variables that determine the transition probabilities, and ϕ_s are vectors of parameters to estimate. When $\underline{z}_{\ell t} = 1 \forall t$ the model boils down to the model of Hamilton (1991) with constant transition probabilities.

¹⁰For example we include fixed effects, oil price shocks and disposable income in the US in $\Delta\hat{Y}_{\ell t}$.

3.2 The Log-likelihood Function and Estimation

We jointly estimate the parameters in equations (4) and (5) by following closely the EM algorithm proposed by Diebold, Lee and Weinbach (1994). Let $\beta_0 = [\beta_0^e \ \beta_0^\psi \ \beta_0^{P_L} \ \beta_0^{y'}]'$, $\beta_1 = [\beta_1^e \ \beta_1^\psi \ \beta_1^{P_L} \ \beta_1^{y'}]'$, and $\theta = [\beta_0' \ \sigma_0 \ \phi_0' \ \beta_1' \ \sigma_1 \ \phi_1']'$. Let $\mathbb{I}_{\ell t}^0$ be the indicator function equal to one if $\xi_{\ell t} = 0$ and zero otherwise (independent of $\xi_{\ell t-1}$); also let $\mathbb{I}_{\ell t}^{00}$ be the indicator function equal to one if $\xi_{\ell t-1} = 0$ and $\xi_{\ell t} = 0$ and zero otherwise; similarly let $\mathbb{I}_{\ell t}^{11}$ be equal to one if $\xi_{\ell t-1} = 1$ and $\xi_{\ell t} = 1$ and zero otherwise; also let $\mathbb{I}_{\ell t}^{10} = 1 - \mathbb{I}_{\ell t}^{00}$ and $\mathbb{I}_{\ell t}^{01} = 1 - \mathbb{I}_{\ell t}^{11}$.

To simplify the notation below let $y_{\ell t} \equiv \Delta \hat{p}_{\ell t}$ be the independent variable in the price equation; let $X_{\ell t} = [\Delta \hat{s}_t \ \Delta \hat{\psi}_{\ell t} \ \Delta \hat{P}_{L t} \ \Delta \hat{Y}'_{\ell t}]'$ be the vector of dependent variables at t in the price equation; and let in general \underline{m}_τ be a $\tau \times 1$ vector of observations of the corresponding variable m for $t = 1, \dots, \tau$. Thus $\underline{y}_{\ell T}$ is the vector with T observations of our independent variable, $\underline{X}_{\ell T}$ is the matrix of explanatory variables in the pricing equation (4) and $\underline{Z}_{\ell T}$ is the matrix of variables in the probability equations (5)—the matrices $y_{\ell t}$, $\underline{y}_{\ell T}$, $X_{\ell t}$, $\underline{X}_{\ell T}$ and $\underline{Z}_{\ell T}$ represent our data after the appropriate transformations, see Section 4.

The contribution of the unit ℓ to the complete-data likelihood function is:¹¹

$$\begin{aligned} L_\ell \left(\underline{y}_{\ell T}, \underline{\xi}_{\ell T}, \underline{X}_{\ell T}, \underline{Z}_{\ell T}; \theta \right) &= \prod_{t=1}^T \mathfrak{F}(y_{\ell t}, \xi_{\ell t} | \underline{y}_{\ell t-1}, \underline{\xi}_{\ell t-1}, \underline{X}_{\ell T}, \underline{z}_{\ell T}; \theta) \\ &= \prod_{t=1}^T \mathfrak{F}(y_{\ell t} | \xi_{\ell t}, \underline{X}_{\ell T}; \beta_0, \beta_1, \sigma_0, \sigma_1) \Pr(\xi_{\ell t} | \xi_{\ell t-1}, z_{\ell t-1}; \phi_0, \phi_1) \end{aligned}$$

where

$$\mathfrak{F}(y_{\ell t} | \xi_{\ell t} = i, \underline{X}_{\ell T}; \beta_i, \sigma_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(y_{\ell t} - X'_{\ell t}\beta_i)^2}{2\sigma_i^2}\right)$$

for $i = 0, 1$.

¹¹It is complete-data likelihood function in the sense that we observe ξ_ℓ as well as the data in all possible states. Of course, the econometrician only observes the prices $\hat{p}_{\ell t}$ and she has to make an inference about the state $\xi_{\ell t}$.

We can conveniently write the contribution of the unit ℓ to the complete-data log-likelihood function in terms of the indicator function as:

$$\begin{aligned} \log L_\ell \left(\underline{y}_{\ell T}, \underline{\xi}_{\ell T}, | \underline{X}_{\ell T}, \underline{Z}_{\ell T}; \theta \right) &= \sum_{t=1}^T \left\{ \mathbb{I}^0 \log \mathfrak{F}(y_{\ell t} | \xi_{\ell t} = 0, \underline{X}_T; \beta_0) \right. \\ &\quad + [1 - \mathbb{I}^0] \log \mathfrak{F}(y_{\ell t} | \xi_{\ell t} = 1, \underline{X}_T; \beta_1) + \mathbb{I}^{00} \log g_{\ell t}^{00} + \mathbb{I}^{10} \log(1 - g_{\ell t}^{00}) \\ &\quad \left. + \mathbb{I}^{11} \log g_{\ell t}^{11} + \mathbb{I}^{01} \log(1 - g_{\ell t}^{11}) \right\} \end{aligned}$$

Now consider a panel data with \mathbb{L}^* automobile lines and assume further that the Markov processes ξ_ℓ are independent across units ℓ . The complete-data log-likelihood function is:

$$\log L \left(\underline{y}_T, \underline{\xi}_T, | \underline{X}_T, \underline{Z}_T; \theta \right) = \sum_{\ell=1}^{\mathbb{L}^*} \log L_\ell \left(\underline{y}_{\ell T}, \underline{\xi}_{\ell T}, | \underline{X}_{\ell T}, \underline{Z}_{\ell T}; \theta \right).$$

Since it is not feasible to construct the complete-data log-likelihood function, the EM algorithm is often employed to maximize the incomplete-data log-likelihood function. The EM algorithm is a two-step iterative procedure to maximize the expected complete-data log-likelihood function conditional upon the observed data. It is initiated by assigning initial probabilities for being in each state. In the first step (the *expectation* step), conditional on the initial guess, inferences on $\xi_{\ell t}$ are obtained using all the information in the sample. These inferences are called the smoothed probabilities. Then, in the second step (*maximization*), the expected complete-data log likelihood is maximized with respect to the parameters of the model. The procedure is iterated until θ converges. See Diebold, Lee and Weinbach (1994) for a detailed description of the EM procedure and the appendix for our implementation. Once the estimates of θ are obtained, we can make inferences about the regime that was more likely to have been in effect in setting the price of a specific automobile line for a given year.

We compute the variance-covariance matrix following the supplemented EM algorithm (SEM) of

Mang and Rubin (1991). The main idea behind SEM is to find the increased variability due to missing information (in our case unobservable regimes), and add it to the complete data variance-covariance matrix, which we find analytically based on the information matrix. The details on computing the variance-covariance matrix, and the SEM algorithm can be found in the Appendix.

4 Data Description and Sources

Our data on automobile imports into the US comes from *Ward's Automotive Yearbook*. We have collected information on automobile imports for the 1980-2004 period. Although *Ward's Automotive Yearbook* has information on more imported automobiles, we restrict our attention to 35 lines. Our choice of automobile models depends on the availability of price and quantity data for the baseline models. Because we would like to analyze the changes in prices of individual goods and link them to macroeconomic trends, we look at the models that have information for at least ten consecutive years. Furthermore, we restrict our choice of models based on the availability of information on the input-sourcing for each model. Gron and Swenson (2000) have shown that accounting for factor-market decisions of firms are important in measuring pass-through. In order to control for marginal costs incurred in different locations, we choose the lines for which we know the input sources and content of production. As a result, we end up with 35 baseline models from seven exporting countries: France, Germany, Italy, Japan, Korea, Sweden and the United Kingdom.

As our dependent variable, we use the manufacturer's suggested retail price (in U.S. dollars) at the port of entry¹². Since we are interested in the pricing decisions of exporting firms, we would like to get prices that are net of any additions that the dealers might charge. Therefore, we do not use the transaction prices. The manufacturer's suggested retail prices do not include destination charges¹³, state or local taxes or optional equipment. However, they include ocean freight and U.S. import duty.

¹²Since we are looking at prices at the port of entry, the pass through coefficient estimates only reflect the pass through at the dock, and not the pass-through to the final consumer prices.

¹³After 1990, Wards Automotive Yearbook reports prices including the destination charges. For those years, we collected the information on destination charges from the *Market Data Book*, and subtracted them from the reported prices.

Ward's Automotive Yearbook provides information also on the physical attributes, segment and sales of each model. The physical attributes include engine specifications (size, horse power, cylinders, etc.) and dimensions (height, weight, length). We use the information on the physical characteristics of the car to adjust the prices for quality differences, and use the quality-adjusted prices in our estimations.¹⁴ We also use the information on physical attributes in addition to prices to categorize the automobiles in different market segments. The automobiles in our sample fall into one of the three segments: small, middle and luxury. This categorical variable helps us calculate the share of sales of each line in its own segment. Moreover, we build a Herfindahl index using the total quantity sold in each segment to measure the market concentration. Finally, we use the data on sales to construct the total market share of exporting countries. As suggested by Feenstra, Gagnon and Knetter (1996) and Bacchetta and van Wincoop (2005), the market share is defined as quantity of exports by a country to the U.S. as a ratio of total new automobile sales in the US.

All the other variables used as regressors in the pricing equation and those in the probability equation are constructed from monthly series so that the information set corresponds to the information set available to the exporter at the time of the announcement of the price. The model year runs from September to October of the following year. Hence, we construct the exchange rate variable as an average of monthly nominal market rates, official rates if market rates are not available (source: International Financial Statistics), over the model year of the automobiles. Our proxy for the exchange rate volatility is the average of monthly squared changes in the log of exchange rate during the previous model year. Inflation variable is the average of annualized inflation rates calculated from consumer price indices (source: International Financial Statistics).

For most of the models in our sample *Ward's Automotive Yearbook* reports a single production location, which is the manufacturer's country of origin. As the marginal cost of those models, we use the monthly manufacturing wage rates of the exporting countries (reported by Bureau of Labor Statistics),

¹⁴To obtain the quality-adjusted prices we regress our original prices against the ratio of horse-power to car weight and eliminate the systematic component.

convert them to dollar terms using the monthly exchange rates, and construct the averages over the model year.

5 Empirical Estimates

In this section we present estimates of the pricing policies used in exports of new cars from England, France, Germany, Italy, Japan, Korea and Sweden to the US. Our sample is a panel data of prices of 35 narrowly defined car models that covers the period 1980-2004. Our strategy in the sequence of models is first to keep constant the transition probabilities as in Hamilton (1991) and discuss the dimensions in which the low pass-through regime differs from the high-pass-through one. Next, we consider the fundamental variables that various theories have identified as important determinants for the choice of currency denomination of exports and the degree of exchange rate pass-through. Hence we allow for time-varying transition probabilities as in Diebold et al. (1994) and discuss the significance of various economic indicators in the light of estimation results.

5.1 Constant Transition Probabilities

As highlighted in equation (4), we allow the optimal export price of the firm to respond to changes in the exchange rate surprise, expected changes in marginal cost, proxied by changes in the wage index of exporter's country, and to expected changes in prices of competitors, proxied by changes in the US CPI of new cars. Moreover, we control for the automobile line specific fixed effects and US disposable income.¹⁵ We constrain the fixed effects and disposable income to be equal across regimes, and focus on the parameter instability in the exchange rate pass-through, marginal cost and industry price index. The transition probabilities (5), as in Hamilton (1991), are constant.¹⁶

The parameter estimates, standard errors and some statistics for the model with constant transition

¹⁵In some alternative specification, we have also controlled for oil price shocks. The results look very similar, and are available upon request.

¹⁶If we include segment dummies that correspond to luxury, medium and small models, instead of a single constant, our conclusions below do not change.

probabilities are reported in Table 1. Table 1 shows that our estimation identifies two distinct regimes that are characterized by three results. First, we obtain two different and highly significant exchange rate pass-through coefficients. While the exchange rate pass-through is 16.13% in the "high-pass-through" (HPT) regime, it is 4.87% in the "low-pass-through" (LPT) one. Second, LPT regime is also characterized by lower sensitivity to changes in the industry price index; one coefficient being almost twice as big as the other (0.5027 vs. 0.9149). Third, in the LPT regime the joint volatility of technology and preference shocks measured by the variance of the shocks is low ($\hat{\sigma}_0^2 = 0.0002$) whereas in the HPT regime it is much higher ($\hat{\sigma}_1^2 = 0.0127$).¹⁷ The estimation results show that the changes in the wage rate are not very different across the two regimes, and it is not significant in the HPT regime. While tests about the existence of two regimes versus one regime are not fully developed in the literature at this time (see discussion in Hamilton 2005), the three significantly different coefficients we identify exposes the instability of the whole export pricing regime, and not only the pass-through coefficient. Furthermore, the likelihood ratio test for the equality of the pass-through coefficients across the two regimes rejects the equality constraint.

One important implication emerges from our first specification. In states of the economy where exporters are faced with a mix of preference and technology shocks with low volatility, they smooth further prices by passing a lower percentage of changes in both, exchange rates and marginal costs; therefore there is a non-linear relation between the volatility of exporters' prices and the volatility of exogenous shocks that they face. Moreover, this implication carries over more general specifications presented below.

Although this specification with constant transition probabilities is illustrative, there are theoretical arguments to believe that the transition probabilities across pricing regimes are not constant over time but vary with macroeconomic and/or microeconomic conditions. We start to explore some of those theoretical arguments in the next subsection.

¹⁷As a reference, business cycle models for the US estimate the variance of technology shocks in the US around 0.00008 and preference shocks around 0.00091. Our estimates of σ s jointly account for both, technology shocks in the exporter's country and preference shocks in the US.

5.2 Economic Factors as Drivers of Transition Probabilities

The theoretical literature on export pricing suggests a diverse set of variables for the determination of the optimal degree of exchange rate pass-through. While a strand of the literature focuses on firm specific and industry specific factors affecting the firms' decisions, other studies focus on country specific factors or macroeconomic conditions. Each of the factors can affect the likelihood of the price being in one of the two regimes. By allowing the transition probabilities be functions of one or more of these factors, we investigate their significance in the export pricing and pass-through decisions. Furthermore, we analyze the role of each of these factors in leading to a decline in the average pass-through.

To clarify the interpretation of our estimates of the conditional probabilities (5), and to shed some light on the relevance of the economic factors for the pass-through, we consider the following. Assume that there is a continuum of mass one of firms exporting to the US; out of that mass of firms, the fraction Λ_t is subject to the “low pass-through” state in t (i.e. $\xi_t = 0$) and a mass $(1 - \Lambda_t)$ is subject to the high pass-through. Recall that g_t^{00} is the transition probability of a firm acting under the low pass-through state in t given that it was in the same state in $t - 1$; and $(1 - g_t^{11}) = g_t^{01}$ is the transition probability of a firm switching from the high pass-through regime in $t - 1$ to the low pass-through regime in t . Thus, in this setting, the evolution of the mass of firms in the low pass-through regime is given by

$$\Lambda_t = \Lambda_{t-1}g_t^{00} + (1 - \Lambda_{t-1})(1 - g_t^{11}), \quad (7)$$

where $g_t^{ii} = \frac{\exp(z'_{\ell t-1}\hat{\phi}_i)}{1 + \exp(z'_{\ell t-1}\hat{\phi}_i)}$. Given the initial condition Λ_{-1} the dynamics of the fraction of firms in the low pass-through regime is driven by the transition probabilities g_t^{00} and g_t^{11} , which in turn, we assume can be driven by economic factors.¹⁸

The major factors that we focus on in our estimations of the pricing equations (4) with time-varying

¹⁸To be precise, we use $g_t^{ii} = 1/L^* \sum_{\ell=1}^{L^*} g_{\ell t}^{ii}$ for $i = 0, 1$; and as initial condition we use the steady-state expression for Λ evaluated with the probabilities estimated for $t = 1$, that is $\Lambda_{-1} = \frac{(1-g_1^{11})}{2-g_1^{00}-g_1^{11}}$. However the initial condition only affects substantially the first couple of years and after that the path of Λ_t is virtually independent of the initial conditions.

transition probabilities (5), g_t^{00} and g_t^{11} , are exchange rate volatility, industry concentration, exporting country's market share and monetary stability. Each of these factors have theoretically been shown to be important in the pass-through decisions in the previous studies. In the following subsections, we briefly review the theoretical arguments, and present the results for each of those factors. In all the specifications discussed below, the coefficients in the pricing equations maintain magnitudes and significance similar to the ones found in the estimation with constant probabilities. Therefore, in the following discussions, we focus our attention to the estimates of the probabilities, and their interpretation.

5.2.1 Industry Specific Factors

The role of product substitutability in the problem of price setting and pass-through have been studied extensively. Some of the seminal papers are Giovannini (1988), Donnenfeld and Zilcha (1991) and Friberg (1998). The main finding common to these papers is that under exchange rate uncertainty, the curvature of the demand and cost functions are important for the choice of currency. Given the common assumptions of constant or decreasing returns to scale for the production technology, high degrees of elasticity of substitution will make the profit function concave. Therefore, if an export good is not very differentiated, the firm can find it more profitable to absorb the exchange rate fluctuations, and not to pass-through much of it. Moreover, the higher the exchange rate uncertainty, the greater the incentive will be to do so.

As an empirical proxy for product substitutability, we consider the Herfindahl index for the US automobile market, which captures the degree of concentration in the market.¹⁹ We examine the effects of market concentration by including the change in Herfindahl index in the probability function.²⁰ The

¹⁹As a second measure we can also use the share of sales of each automobile line in its own segment (small, medium or luxury car). We prefer to focus on the results with the Herfindahl index since towards the end of our sample some automobiles are both imported and produced in the US. Having information only on the imported quantities, our measure of the segment share will be underestimated. The results from this specification show that segment share is significant at 10% in the LPT regime.

²⁰Including the level of the Herfindahl index in the probability function created problems in the convergence of the SEM algorithm. Therefore, we examine the results for the difference of the variable.

second column of estimates in Table 1 shows that the market concentration is a statistically significant determinant of both transition probabilities $g_{\ell t}^{00}$ and $g_{\ell t}^{11}$.

To investigate the implication of the transition probabilities for the propensity to be in the low pass-through regime and the average exchange rate pass-through, we construct the estimated fraction of firms in the LPT regime throughout the sample using equation (7). Panel B of Figure 1 shows the evolution of the fraction of firms, Λ . We also plot the level and the changes of the Herfindahl index. Figure 2 shows that the Herfindahl index for the automobile industry shows a clear downward trend between 1981 and 1991, going down from 0.026 to 0.011. After 1991, the industry seems to have become slightly less concentrated, as the Herfindahl index displays a non-monotonic upward trend. Panel B of Figure 1 shows that the fraction of firms in the LPT regime is on average higher after 1990, and the evolution of the fraction follows the changes in the Herfindahl closely, as shown in Panel B of Figure 2. This is in line with the theories that suggest that higher market concentration (implying higher substitutability) should lead the firms to pass-through less of the exchange rate fluctuations. Therefore, we can infer that the higher market concentration in the post-1990 sample, has led the pass-through to be lower on average in the same period.

5.2.2 Country Specific Factors

Secondly, we consider the total market share of an exporting country as a factor in the determination of the pricing policies. The importance of this factor in the presence of strategic interactions, has been studied by Feenstra, Gagnon and Knetter (1996), Bodnar, Dumas and Marston (2002) and Bacchetta and van Wincoop (2005). These studies highlight the fact that high market share implies that the firms from a particular exporting country do not face much competition from firms that have not experienced similar cost shocks. Therefore, given a certain level of demand in the destination country, the firms can pass-through more of the fluctuations of the exchange rate.

The third column of Table 1 shows that the country share variable is a significant determinant

of both of the conditional probabilities, $g_{\ell t}^{00}$ and $g_{\ell t}^{11}$. The plots of unconditional probabilities for each country, constructed using equation (7), are shown in Figure 3. For most of the countries, there is a systematic positive relationship between the market share and the fraction of firms being in the LPT regime. The exception is Japan before 1995, where an increase in Japan's market share is associated with a decrease in the fraction of Japanese firms in the LPT regime. These results resemble the empirical findings in Feenstra, Gagnon and Knetter (1996), who show that pass-through increases with country market share only when market share is already large, and decreases with market share when it is small. Since all countries, except for Japan, have small shares of the US automobile market, the propensity to choose a low degree of pass-through decreases for the firms in those countries. The implications for the average pass-through can be drawn by looking at the total fraction of firms in the LPT regime. Panel (D) in Figure 1 shows a downward trend in the estimated mass of firms in the LPT regime up until 2000. These results may be driven by the Japanese country share dynamics, given their dominant share in the market.

5.2.3 Macroeconomic Factors

The last factors that we consider relate to monetary and macroeconomic stability in the importing and exporting countries. Taylor (2000) notes that stable inflation affects the degree to which the firms pass-through the fluctuations in the exchange rate to their prices by reducing their pricing power. Similarly, Devereux, Engel and Storgaard (2004) show, in a general equilibrium framework, that the firms optimally set prices in the currency of the country that has more stable money growth. Hence, if the importing country has relatively low and stable inflation rates, more exporting firms will set their prices in the importing country's currency, and as a result, the importing country will experience a lower pass-through.²¹ To empirically evaluate the importance of monetary stability, we include inflation rates

²¹In a set up similar to Devereux et al. (2005), Goldberg and Tille (2005) contrast the role of that macroeconomic conditions to industry specific features for the firms' optimal choice of currency. They show that macroeconomic variability matters for the firms' decisions if their products are highly differentiated. In industries with high elasticities of demand, the firms tend to herd together in the choice of currency rather than basing their decisions on macroeconomic conditions.

of both countries as explanatory variables in the probability functions as well as a measure of the volatility of the exchange rate.

The column “inflation” in Table 1 shows the results for the specification with transition probabilities as functions of the US inflation rate and the inflation rate in the exporter country. The last column in Table 1 shows our estimates for the model with the volatility of the exchange rate.

While the variable US inflation (coefficients ϕ_{02} and ϕ_{12}) is significant in both transition probabilities, the inflation rate of the exporting country is not significant. The volatility of the exchange rate, is also highly significant in the high pass-through regime. Figure 1 shows that inflation and the volatility of the exchange rate imply an upward trend for the fraction of firms in the low pass-through regime. This is consistent with the idea that increased monetary stability in the US has lead the exporters to pass-through less of the exchange rate fluctuations, contributing to the decline in the pass-through.

6 Implications for the Exchange Rate Pass-Through Dynamics

To study the implications of our estimates for the evolution of the average exchange rate pass-through in the automobile industry, we aggregate the individual prices in a price index. Consider the price index of imported cars

$$\hat{P}_t = \sum_{\ell} \omega_{\ell} \hat{p}_{\ell t}$$

where ω_{ℓ} is the weight associated to the car model ℓ and $\hat{p}_{\ell t}$ follows the pricing policy 4. Let $\mathbb{I}_{\ell t}^0$ be an indicator function equal to one if the firm ℓ is in the low pass-through regime in period t and zero otherwise. From the price index and the pricing policy (4) it follows that the exchange rate pass-through coefficient is

$$\eta_t \equiv \frac{\partial \hat{P}_t}{\partial \text{Exch. Rate}_t} = \sum_{\ell} \omega_{\ell} \left[\mathbb{I}_{\ell t}^0 \beta_0^s + (1 - \mathbb{I}_{\ell t}^0) \beta_1^s \right].$$

Our methodology provides as a byproduct an estimate of the probability of $\mathbb{I}_{\ell t}^0 = 1$, this is the

smoothed probability $\mathbb{P}_{\ell t}^0$. This estimate is conditional on the full information contained in the sample.²² Using the smoothed probabilities, we can compute the expected pass-through coefficient at time t , conditional on the full information in the sample, as

$$\mathbb{E}_{\Omega_T} \eta_t = \sum_{\ell} \omega_{\ell} \left[\mathbb{P}_{\ell t}^0 \beta_0^s + (1 - \mathbb{P}_{\ell t}^0) \beta_1^s \right].$$

Note that, different from the rolling regression approach, our inference about the exchange rate pass-through in t is based on the full sample and not on a sub-sample of the data. Also note that, our calculation of the fraction of firms in the low pass-through regime Λ_t (in the previous section), based on the transition probabilities, uses information up to $(t-1)$ for the explanatory variable in the probability equations—see equation (5).

Table 2 presents a general specification that includes market concentration, exporting country’s market share and the US inflation as determinants of the transition probabilities. We estimate the exchange rate pass-through based on this general specification. Figure 2 shows the evolution of the exchange rate pass-through based on the estimates from our general specification in Table 2. It shows a non-monotonic downward trend with the highest pass-through coefficient of 13.4% in 1987 and the lowest pass-through coefficient of 6.4% in 2001.

7 Conclusions

[To be written]

²²To be clear, $\mathbb{P}_{\ell t}^0$ is the probability that the observed price $\hat{p}_{\ell t}$ comes from the low pass-through regime. That probability is based on the full information contained in the data from $t = 1$ to $t = T$.

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8 Technical Appendix: Estimation Procedure

This note assumes familiarity with the regime switching model with time-varying transition probabilities of [?]. These notes are focused on the ‘M’ step of the EM algorithm.

8.1 Likelihood Function

We adapt the regime switching algorithm of [?] as follows. Let ℓ be an index over car lines, assume $\ell = 1, 2, \dots, N$; let t be a time index, for simplicity assume $t = 1, 2, \dots, T$ for each line; and let $s_{\ell t}$ be a line-specific, first-order, two state Markov process. Note that there is a collection of N independent Markov processes, one per car line.

Consider a car line ℓ . The Gaussian regime-switching model for its pricing process is:

$$y_{\ell t} = x'_{\ell t} \beta_{s_t} + \varepsilon_{\ell t} \quad (8)$$

where $\varepsilon_{\ell t} \sim i.i.d. N(0, \sigma_{s_{\ell t}})$ and $y_{\ell t}$ is the change in the logarithm of the Hedonic price; $x_{\ell t}$ are control variables and the state variable $S_{\ell t}$ is unobserved and given by

$$S_{\ell t} = \begin{cases} 0 & \text{if } \eta_{\ell t} < z'_{\ell t-1} \phi_{s_{\ell t-1}} \\ 1 & \text{if } \eta_{\ell t} \geq z'_{\ell t-1} \phi_{s_{\ell t-1}} \end{cases}$$

where $\eta_{\ell t}$ follows a logistic distribution. Thus the transition probabilities are:

$$\begin{aligned} p_{\ell t}^{00} &= P(s_{\ell t} = 0 | s_{\ell t-1} = 0, z_{\ell t-1}; \phi_0) = \Phi(z'_{\ell t-1} \phi_0) \\ p_{\ell t}^{11} &= P(s_{\ell t} = 1 | s_{\ell t-1} = 1, z_{\ell t-1}; \phi_1) = \Phi(z'_{\ell t-1} \phi_1) \\ p_{\ell t}^{01} &= 1 - p_{\ell t}^{00} \\ p_{\ell t}^{10} &= 1 - p_{\ell t}^{11} \end{aligned}$$

with $\Phi(\cdot) = \exp(z'_{\ell t-1} \phi_i) / (1 + \exp(z'_{\ell t-1} \phi_i))$, for $i = 0, 1$.

For $i = 0, 1$ the density function of $y_{\ell t}$ conditional on $x_{\ell t}$ and $s_{\ell t}$ is

$$f(y_{\ell t} | x_{\ell t}, s_{\ell t} = i; \beta_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(y_{\ell t} - x'_{\ell t} \beta_i)^2}{2\sigma_i^2}\right)$$

Different from [?] we follow Hamilton (1994, p. 122) in that we maximize the log-likelihood function conditional on the first observation, as opposed to the exact log-likelihood function; that is, we assume that the first observation $y_{\ell 1}$ is known with certainty to come from the regime i . This greatly simplifies the estimation.

Let $\theta = [\beta'_0 \beta'_1 \phi'_0 \phi'_1 \sigma_0 \sigma_1]'$, then the contribution to the conditional likelihood function from the T observations corresponding to the unit ℓ is

$$\begin{aligned} L_{\ell}(\theta) &= \prod_{t=2}^T f(y_{\ell t}, s_{\ell t} | \underline{y}_{\ell t-1}, \underline{s}_{\ell t-1}, \underline{z}_{\ell T}; \theta) \\ &= \prod_{t=2}^T f(y_{\ell t} | s_{\ell t}; \beta_0, \beta_1, \sigma_0, \sigma_1) P(s_{\ell t} | s_{\ell t-1}, z_{\ell t-1}; \phi_0, \phi_1) \end{aligned}$$

Note that the conditional maximum likelihood regards the first observation of each line $y_{\ell 1}$ as deterministic. This assumption greatly simplifies the computations.

The contribution of the unit ℓ to the complete-data conditional log-likelihood function in terms of the indicator function is:

$$\begin{aligned} \log L_\ell(\theta) = & \sum_{t=2}^T \left\{ I(s_{\ell t} = 0, s_{\ell t-1} = 0) [\log f(y_{\ell t} | s_{\ell t} = 0; \beta_0) + \log p_{\ell t}^{00}] \right. \\ & + I(s_{\ell t} = 1, s_{\ell t-1} = 0) [\log f(y_{\ell t} | s_{\ell t} = 1; \beta_1) + \log(1 - p_{\ell t}^{00})] \\ & + I(s_{\ell t} = 1, s_{\ell t-1} = 1) [\log f(y_{\ell t} | s_{\ell t} = 1; \beta_1) + \log p_{\ell t}^{11}] \\ & \left. + I(s_{\ell t} = 0, s_{\ell t-1} = 1) [\log f(y_{\ell t} | s_{\ell t} = 0; \beta_0) + \log(1 - p_{\ell t}^{11})] \right\} \end{aligned}$$

or

$$\begin{aligned} \log L_\ell(\theta) = & \sum_{t=2}^T \left\{ I(s_{\ell t} = 0) \log f(y_{\ell t} | s_{\ell t} = 0; \beta_0) + I(s_{\ell t} = 1) \log f(y_{\ell t} | s_{\ell t} = 1; \beta_1) \right. \\ & + I(s_{\ell t} = 0, s_{\ell t-1} = 0) \log p_{\ell t}^{00} + I(s_{\ell t} = 1, s_{\ell t-1} = 0) \log(1 - p_{\ell t}^{00}) \\ & \left. + I(s_{\ell t} = 1, s_{\ell t-1} = 1) \log p_{\ell t}^{11} + I(s_{\ell t} = 0, s_{\ell t-1} = 1) \log(1 - p_{\ell t}^{11}) \right\} \end{aligned} \quad (9)$$

The complete-data conditional log-likelihood function of all units is

$$L(\theta) = \sum_{\ell=1}^N \log L_\ell(\theta)$$

8.2 Expected complete-data log-likelihood.

The EM algorithm is a robust procedure to maximize the incomplete-data log-likelihood function based on the expected complete-data log-likelihood. Thus the expected complete-data conditional log-likelihood function, conditional on the observed data, is:

$$EL(\theta) = \sum_{\ell=1}^N E \log L_\ell(\theta)$$

To save notation, let $\mathbb{P}_{\ell t}(i) \equiv P(s_{\ell t} = i | \underline{x}_{\ell T}, \underline{y}_{\ell T}, \underline{z}_{\ell T}; \theta)$ and $\mathbb{P}_{\ell t}(i, j) \equiv P(s_{\ell t} = i, s_{\ell t-1} = j | \underline{x}_{\ell T}, \underline{y}_{\ell T}, \underline{z}_{\ell T}; \theta)$ with $i=0,1$ and $j=0,1$ denote the smoothed probabilities. Then

$$\begin{aligned} E \log L(\theta) = & \sum_{\ell=1}^N \sum_{t=2}^T \left\{ \mathbb{P}_{\ell t}(0) \log f(y_{\ell t} | s_{\ell t} = 0; \beta_0) + \mathbb{P}_{\ell t}(1) \log f(y_{\ell t} | s_{\ell t} = 1; \beta_1) \right. \\ & + \mathbb{P}_{\ell t}(0, 0) \log p_{\ell t}^{00} + \mathbb{P}_{\ell t}(1, 0) \log(1 - p_{\ell t}^{00}) \\ & \left. + \mathbb{P}_{\ell t}(1, 1) \log p_{\ell t}^{11} + \mathbb{P}_{\ell t}(0, 1) \log(1 - p_{\ell t}^{11}) \right\} \end{aligned} \quad (10)$$

To obtain the first-order conditions we follow as closely [?], however we depart from [?] in one aspect. Since the first-order conditions with respect to ϕ s involve a non-linear equation in parameters, [?] approximate the first-order conditions with respect to ϕ s with a first-order Taylor approximation. Instead, we approximate the terms involving p^{00} and p^{11} in the log-likelihood function (10) with a second-order Taylor approximation and we maximize the approximated log-likelihood function. Perhaps surprisingly, we obtain the exact same first-order conditions than [?]. Thus the two approaches are equivalent.

Accordingly, consider the following Taylor approximation of the terms involving $p_{\ell t}^{ii}(\phi^{(k)})$ and for $i = 0, 1$ in the iteration k around the parameter values of the iteration $k - 1$ —that is, all derivatives in the approximation are evaluated at values found in the previous iteration²³. For simplicity set $i = 0$ and assume that the are W variables in the function Φ , that is, ϕ_0 and ϕ_1 are $W \times 1$ vectors, then

$$\begin{aligned} \log(p_{\ell t}^{00}(\phi^{(k)})) &\approx \log(p_{\ell t}^{00}(\phi^{(k-1)})) \\ &+ \sum_{w=1}^W z_{\ell t-1}(w)(1 - p_{\ell t}^{00}(\phi^{(k-1)}))(\phi_{0w}^{(k)} - \phi_{0w}^{(k-1)}) \\ &- \frac{1}{2}p_{\ell t}^{00}(\phi^{(k-1)})(1 - p_{\ell t}^{00}(\phi^{(k-1)})) \sum_{w=1}^W \sum_{\iota=1}^W z_{\ell t-1}(w)z_{\ell t-1}(\iota)(\phi_{0w}^{(k)} - \phi_{0w}^{(k-1)})(\phi_{0\iota}^{(k)} - \phi_{0\iota}^{(k-1)}) \end{aligned} \quad (11)$$

and

$$\begin{aligned} \log(1 - p_{\ell t}^{00}(\phi^{(k)})) &\approx \log(1 - p_{\ell t}^{00}(\phi^{(k-1)})) \\ &- \sum_{w=1}^W z_{\ell t-1}(w)p_{\ell t}^{00}(\phi^{(k-1)})(\phi_{0w}^{(k)} - \phi_{0w}^{(k-1)}) \\ &- \frac{1}{2}p_{\ell t}^{00}(\phi^{(k-1)})(1 - p_{\ell t}^{00}(\phi^{(k-1)})) \sum_{w=1}^W \sum_{\iota=1}^W z_{\ell t-1}(w)z_{\ell t-1}(\iota)(\phi_{0w}^{(k)} - \phi_{0w}^{(k-1)})(\phi_{0\iota}^{(k)} - \phi_{0\iota}^{(k-1)}). \end{aligned} \quad (12)$$

Similarly we approximate the terms involving $p_{\ell t}^{11}(\phi^{(k)})$.

We derive the first-order conditions from the function (10) with the the terms involving $p_{\ell t}^{00}$ and $p_{\ell t}^{11}$ replaced by their second-order Taylor approximations (11) and (12). To obtain the first-order conditions first consider the first-order condition with respect to the j -th element of β_i ; let's denote with $x_{\ell t}(j)$ the associated variable. In k -th iteration consider $\theta^{(k-1)}$ as given from the previous iteration; thus we

²³Note that

$$\begin{aligned} \log p_{\ell t}^{00} &= z'_{\ell t-1}\phi_0 - \log(1 + \exp(z'_{\ell t-1}\phi_0)) \\ \log(1 - p_{\ell t}^{00}) &= -\log(1 + \exp(z'_{\ell t-1}\phi_0)) \end{aligned}$$

then the first derivatives are

$$\begin{aligned} \frac{d \log p_{\ell t}^{00}}{d\phi_{0w}} &= z_{\ell t-1}(w)(1 - p_{\ell t}^{00}) \\ \frac{d \log(1 - p_{\ell t}^{00})}{d\phi_{0w}} &= -z_{\ell t-1}(w)p_{\ell t}^{00} \end{aligned}$$

and the second derivatives are

$$\begin{aligned} \frac{d^2 \log p_{\ell t}^{00}}{d\phi_{0w}d\phi_{0j}} &= -z_{\ell t-1}(w)z_{\ell t-1}(j)p_{\ell t}^{00}(1 - p_{\ell t}^{00}) \\ \frac{d^2 \log(1 - p_{\ell t}^{00})}{d\phi_{0w}d\phi_{0j}} &= -z_{\ell t-1}(w)z_{\ell t-1}(j)p_{\ell t}^{00}(1 - p_{\ell t}^{00}) \end{aligned}$$

have ²⁴

$$\begin{aligned} \frac{dEL(\theta)}{d\beta_{ij}} &= \sum_{\ell=1}^N \sum_{t=2}^T \mathbb{P}_{\ell t}(i) \frac{d \log f(y_{\ell t} | s_{\ell t} = i; \beta_i)}{d\beta_{ij}} \\ &= \sum_{\ell=1}^N \sum_{t=2}^T \mathbb{P}_{\ell t}(i) \left(\frac{y_{\ell t} - x'_{\ell t} \beta_i}{\sigma_i^2} \right) x_{\ell t}(j) = 0 \end{aligned}$$

Thus the first-order conditions with respect to β 's can be written as

$$\sum_{\ell=1}^N \sum_{t=2}^T \mathbb{P}_{\ell t}(i) (y_{\ell t} - x'_{\ell t} \beta_i) x_{\ell t}(j) = 0 \quad (13)$$

To save notation let X be the $NT \times J$ stacked matrix of explanatory variables, assuming J explanatory variables and a total of NT observations; also let $X(j)$ be the j -th column of X ; let Y be the stacked vector of $NT \times 1$ observations; let P_i be the $(NT \times 1)$ stacked vector of marginal smoothed probabilities $\mathbb{P}_{\ell t}(i)$. Since the summation in (13) starts with $t = 2$ it is convenient to define \tilde{P}_i equal to P_i but with the first observation of each line replaced by zero—this simplifies the matrix notation below.

To facilitate the exposition let $A \odot B$ denote the element by element product of the two vectors A and B ; thus $C = A \odot B$ has the same length than the vectors A and B ²⁵.

Let²⁶

$$\begin{aligned} \tilde{X}_i &= [\tilde{P}_i \odot X(1) \quad \tilde{P}_i \odot X(2) \cdots \tilde{P}_i \odot X(J)], \\ \tilde{Y}_i &= \tilde{P}_i \odot Y \end{aligned}$$

and let $\beta_i^{(k)}$ be the $J \times 1$ vector of coefficients associated to the state $s = i$ estimated in the iteration k . It is straightforward to verify that the first-order condition for the j -th element of $\beta_i^{(k)}$ described in (13) can be written as:

$$0 = [\tilde{Y}_i - \tilde{X}_i \beta_i^{(k)}]' X(j)$$

Stacking the J first-order conditions in an $J \times 1$ vector we have

$$\begin{aligned} \mathbf{0}_{J \times 1} &= \left[[\tilde{Y}_i - \tilde{X}_i \beta_i^{(k)}]' [X(1) \quad X(2) \cdots X(J)] \right]' \\ &= X' [\tilde{Y}_i - \tilde{X}_i \beta_i^{(k)}] \end{aligned}$$

²⁴To be clear, consider for example $i = 0$. Note that

$$\log f(y_{\ell t} | s_{\ell t} = 0; \beta_0) = -\frac{1}{2} \log(2\pi\sigma_0^2) - \frac{(y_{\ell t} - x'_{\ell t} \beta_0)^2}{2\sigma_0^2}$$

then:

$$\frac{d \log f(y_{\ell t} | s_{\ell t} = 0; \beta_0)}{d\beta_{0j}} = \left(\frac{y_{\ell t} - x'_{\ell t} \beta_0}{\sigma_0^2} \right) x_{\ell t}(j)$$

and

$$\frac{d \log f(y_{\ell t} | s_{\ell t} = 0; \beta_0)}{d\sigma_0^2} = -\frac{1}{2\sigma_0^2} + \frac{(y_{\ell t} - x'_{\ell t} \beta_0)^2}{2(\sigma_0^2)^2}$$

²⁵In MATLAB this corresponds to $C = A .* B$

²⁶Note: \tilde{X}_i is the $NT \times J$ matrix with each row formed by the product of the t -th row of X times the t -th row (a scalar) of the vector \tilde{P}_i ; similarly \tilde{Y}_i is the $NT \times 1$ vector with each row formed by the product of the t -th row of Y times the t -th row of the vector \tilde{P}_i .

Thus

$$\hat{\beta}_i^{(k)} = [X' \tilde{X}_i]^{-1} X' \tilde{Y}_i \quad (14)$$

The first-order conditions with respect to σ_i^2 imply

$$\sum_{\ell=1}^N \sum_{t=2}^T \mathbb{P}_{\ell t}(i) \left(\frac{(y_{\ell t} - x'_{\ell t} \beta_i)^2}{\sigma_i^2} - 1 \right) = 0 \quad (15)$$

Using the estimated vector $\hat{\beta}_i^{(k)}$ let ξ_i^2 be a $NT \times 1$ vector with each row given the $(y_{\ell t} - x'_{\ell t} \hat{\beta}_i^{(k)})^2$ and let $\mathbf{1}_{NT}$ be a vector of $NT \times 1$ ones. The first-order condition (15) can be written as

$$0 = \tilde{P}'_i \xi_i^2 - \sigma_i^2 \tilde{P}'_i \mathbf{1}_{NT},$$

thus

$$\hat{\sigma}_i^{2(k)} = \frac{\tilde{P}'_i \xi_i^2}{\tilde{P}'_i \mathbf{1}_{NT}}, \quad (16)$$

Recall that we use the a second-order Taylor expansions (11) and (12) in the log-likelihood function (10) to approximate the terms involving p^{00} and p^{11} . Consider the first-order conditions with respect to the v -th element of ϕ_0 :

$$\frac{d \log L(\cdot)}{d \phi_{0v}} = \sum_{\ell=1}^N \sum_{t=2}^T \left\{ \mathbb{P}_{\ell t}(0, 0) \frac{d \log p_{\ell t}^{00}}{d \phi_{0v}} + \mathbb{P}_{\ell t}(1, 0) \frac{d \log(1 - p_{\ell t}^{00})}{d \phi_{0v}} \right\} = 0$$

or

$$0 = \sum_{\ell=1}^N \sum_{t=2}^T \left\{ \mathbb{P}_{\ell t}(0, 0) z_{\ell t-1}(v) \left\{ (1 - p_{\ell t}^{00}) - p_{\ell t}^{00} (1 - p_{\ell t}^{00}) \sum_{w=1}^W z_{\ell t-1}(w) (\phi_{0w}^{(k)} - \phi_{0w}^{(k-1)}) \right\} \right. \\ \left. - \mathbb{P}_{\ell t}(1, 0) z_{\ell t-1}(v) \left\{ p_{\ell t}^{00} + p_{\ell t}^{00} (1 - p_{\ell t}^{00}) \sum_{w=1}^W z_{\ell t-1}(w) (\phi_{0w}^{(k)} - \phi_{0w}^{(k-1)}) \right\} \right\} \quad (17)$$

Using the notation above, assume that there are W variables in the probability function Φ , thus Z is an $N(T-1) \times W$ matrix of lagged variables—it is of length $N(T-1)$ since it contains lags. Let \tilde{Z} be a $NT \times W$ matrix where the first element of each line ℓ is zero and the remaining elements contain the corresponding elements of Z . Let \tilde{P}_{ij} be the $NT \times 1$ stacked vector of smoothed joint probabilities $\mathbb{P}_{\ell t}(i, j)$ for $i = 0, 1$ and $j = 0, 1$ with the first element of each unit ℓ is replaced by zero—this helps to keep the $NT \times 1$ dimension of the vector despite that the summation in (17) starts in $t = 2$. Let p^{00} be the $NT \times 1$ stacked vector of conditional probabilities.

Let

$$\begin{aligned} Q_0(0) &\equiv \tilde{P}(0, 0) - [\tilde{P}(0, 0) + \tilde{P}(1, 0)] \odot p^{00} \\ Q_0(1) &\equiv [\tilde{P}(0, 0) + \tilde{P}(1, 0)] \odot p^{00} \odot (\mathbf{1}_{NT} - p^{00}) \odot \tilde{Z}(1) \\ Q_0(2) &\equiv [\tilde{P}(0, 0) + \tilde{P}(1, 0)] \odot p^{00} \odot (\mathbf{1}_{NT} - p^{00}) \odot \tilde{Z}(2) \\ &\vdots \\ Q_0(W) &\equiv [\tilde{P}(0, 0) + \tilde{P}(1, 0)] \odot p^{00} \odot (\mathbf{1}_{NT} - p^{00}) \odot \tilde{Z}(W) \end{aligned}$$

Note that all Q s are $NT \times 1$ vectors. The first-order condition (17) can be written as:

$$\begin{aligned} \tilde{Z}(w)'Q_0(0) &= \tilde{Z}(w)'Q_0(1)(\phi_{01}^k - \phi_{01}^{k-1}) \\ &\quad + \tilde{Z}(w)'Q_0(2)(\phi_{02}^k - \phi_{02}^{k-1}) + \dots + \tilde{Z}(w)'Q_0(W)(\phi_{0W}^k - \phi_{0W}^{k-1}) \end{aligned}$$

Stacking all W first-order conditions associated to ϕ_0 and arranging in matrix notation we have:

$$\begin{bmatrix} \tilde{Z}(1)'Q_0(0) \\ \tilde{Z}(2)'Q_0(0) \\ \vdots \\ \tilde{Z}(W)'Q_0(0) \end{bmatrix} = \begin{bmatrix} \tilde{Z}(1)'Q_0(1) & \tilde{Z}(1)'Q_0(2) & \dots & \tilde{Z}(1)'Q_0(W) \\ \tilde{Z}(2)'Q_0(1) & \tilde{Z}(2)'Q_0(2) & \dots & \tilde{Z}(2)'Q_0(W) \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{Z}(W)'Q_0(1) & \tilde{Z}(W)'Q_0(2) & \dots & \tilde{Z}(W)'Q_0(W) \end{bmatrix} \begin{bmatrix} \phi_{01}^{(k)} - \phi_{01}^{(k-1)} \\ \phi_{02}^{(k)} - \phi_{02}^{(k-1)} \\ \vdots \\ \phi_{0W}^{(k)} - \phi_{0W}^{(k-1)} \end{bmatrix}$$

or

$$\tilde{Z}'Q_0(0) = \tilde{Z}'Q_0[\phi_0^{(k)} - \phi_0^{(k-1)}]$$

where $Q_0 \equiv [Q_0(1) \ Q_0(2) \ \dots \ Q_0(W)]$. Then

$$\phi_0^{(k)} = \phi_0^{(k-1)} + [\tilde{Z}'Q_0]^{-1} \tilde{Z}'Q_0(0). \quad (18)$$

Similarly, for $\phi_1^{(k)}$ let

$$\begin{aligned} Q_1(0) &\equiv \tilde{P}(1, 1) - [\tilde{P}(1, 1) + \tilde{P}(0, 1)] \odot p^{11} \\ Q_1(1) &\equiv [\tilde{P}(1, 1) + \tilde{P}(0, 1)] \odot p^{11} \odot (\mathbf{1}_{NT} - p^{11}) \odot \tilde{Z}(1) \\ Q_1(2) &\equiv [\tilde{P}(1, 1) + \tilde{P}(0, 1)] \odot p^{11} \odot (\mathbf{1}_{NT} - p^{11}) \odot \tilde{Z}(2) \\ &\quad \vdots \\ Q_1(W) &\equiv [\tilde{P}(1, 1) + \tilde{P}(0, 1)] \odot p^{11} \odot (\mathbf{1}_{NT} - p^{11}) \odot \tilde{Z}(W) \end{aligned}$$

and

$$\phi_1^{(k)} = \phi_1^{(k-1)} + [\tilde{Z}'Q_1]^{-1} \tilde{Z}'Q_1(0) \quad (19)$$

where $Q_1 \equiv [Q_1(1) \ Q_1(2) \ \dots \ Q_1(W)]$.

In the MATLAB code we use (14), (16), (18) and (19) to find β_i , σ_i , ϕ_i for $i=0,1$.

8.3 Constrained Estimator

Next we consider a model with some coefficients in the pricing equation constrained to be equal across regimes; and/or equal variance of the error term in the pricing equation.

Let $\beta_0^c = [\beta_{01} \dots \beta_{0J^*} \ \beta_{c1} \dots \beta_{cJ^c}]'$ and $\beta_1^c = [\beta_{11} \dots \beta_{1J^*} \ \beta_{c1} \dots \beta_{cJ^c}]'$ be the vectors of coefficients arranged in a convenient way. Note that there are J^* state-varying coefficients and J^c coefficients constrained to be equal across regimes. The pricing policy is

$$y_{\ell t} = \begin{cases} x'_{\ell t} \beta_0^c + \varepsilon_{\ell t} & \text{if } s_{\ell t} = 0 \\ x'_{\ell t} \beta_1^c + \varepsilon_{\ell t} & \text{if } s_{\ell t} = 1 \end{cases} \quad (20)$$

with $\varepsilon_{\ell t} \sim i.i.d.N(0, \sigma_{s_{\ell t}})$. Using the notation described above and using the pricing equations (20),

consider the following $2J^* + J^c$ first order conditions with respect to β s

$$\begin{aligned} 0 &= [\tilde{Y}_0 - \tilde{X}_0 \beta_0^{c(k)}]' X(j^*) & j^* &= 1 \dots J^* \\ 0 &= [\tilde{Y}_1 - \tilde{X}_1 \beta_1^{c(k)}]' X(j^*) & j^* &= 1 \dots J^* \\ 0 &= [(\sigma_0^2)^{-1} [\tilde{Y}_0 - \tilde{X}_0 \beta_0^{c(k)}] + (\sigma_1^2)^{-1} [\tilde{Y}_1 - \tilde{X}_1 \beta_1^{c(k)}]]' X(j^c) & j^c &= J^* + 1 \dots J^* + J^c \end{aligned}$$

Note that if we impose the restriction $\sigma_0 = \sigma_1$, the last set of first-order conditions above are independent of σ —the σ s drop out in the third set of conditions above. However, if we let $\sigma_0 \neq \sigma_1$ then the model is not linear these parameters. Assume $\sigma_0 \neq \sigma_1$ and use $\sigma_0^{(k-1)}$ and $\sigma_1^{(k-1)}$ from the $k-1$ iteration to compute the following matrixes. Below, we'll describe how to obtain the full set of parameters.

Let $\beta_c^{(k)} = [\beta_{01} \dots \beta_{0J^*} \beta_{11} \dots \beta_{1J^*} \beta_{c1} \dots \beta_{cJ^c}]'$ be the full vector of β parameters, this is a vector of length $2J^* + J^c$. Let

$$\begin{aligned} \tilde{X}_0 &= [\tilde{P}_0 \odot X(1) \dots \tilde{P}_0 \odot X(J^*) \quad \mathbf{0}_{NT \times J^*} \quad \tilde{P}_0 \odot X(J^* + 1) \dots \tilde{P}_0 \odot X(J^* + J^c)] \\ \tilde{X}_1 &= [\mathbf{0}_{NT \times J^*} \quad \tilde{P}_1 \odot X(1) \dots \tilde{P}_1 \odot X(J^*) \quad \tilde{P}_1 \odot X(J^* + 1) \dots \tilde{P}_1 \odot X(J^* + J^c)] \end{aligned}$$

Thus we can *equivalently* write the first-order conditions above in terms of β_c as opposed to β_0^c and β_1^c as follows:

$$\begin{aligned} 0 &= [\tilde{Y}_0 - \tilde{X}_0 \beta_c^{(k)}]' X(j^*) & j^* &= 1 \dots J^* \\ 0 &= [\tilde{Y}_1 - \tilde{X}_1 \beta_c^{(k)}]' X(j^*) & j^* &= 1 \dots J^* \\ 0 &= \left[[\tilde{Y}_0 - \tilde{X}_0 \beta_c^{(k)}] + \frac{\sigma_0^2}{\sigma_1^2} [\tilde{Y}_1 - \tilde{X}_1 \beta_c^{(k)}] \right]' X(j^c); & j^c &= J^* + 1 \dots J^* + J^c \end{aligned}$$

Let $X_{J^*} \equiv [X(1) \ X(2) \dots X(J^*)]$ and $X_{J^c} \equiv [X(J^* + 1) \ X(J^* + 2) \dots X(J^* + J^c)]$. Stacking the first-order conditions we get

$$\begin{aligned} \mathbf{0}_{1 \times J^*} &= [\tilde{Y}_0 - \tilde{X}_0 \beta_c^{(k)}]' X_{J^*} \\ \mathbf{0}_{1 \times J^*} &= [\tilde{Y}_1 - \tilde{X}_1 \beta_c^{(k)}]' X_{J^*} \\ \mathbf{0}_{1 \times J^c} &= \left[[\tilde{Y}_0 - \tilde{X}_0 \beta_c^{(k)}] + \frac{\sigma_0^2}{\sigma_1^2} [\tilde{Y}_1 - \tilde{X}_1 \beta_c^{(k)}] \right]' X_{J^c} \end{aligned}$$

or rearranging in a $2J^* + J^c$ vector ²⁷

$$\begin{bmatrix} X'_{J^*} \tilde{Y}_0 \\ X'_{J^*} \tilde{Y}_1 \\ X'_{J^c} [\tilde{Y}_0 + \frac{\sigma_0^2}{\sigma_1^2} \tilde{Y}_1] \end{bmatrix} = \begin{bmatrix} X'_{J^*} \tilde{X}_0 \\ X'_{J^*} \tilde{X}_1 \\ X'_{J^c} [\tilde{X}_0 + \frac{\sigma_0^2}{\sigma_1^2} \tilde{X}_1] \end{bmatrix} \beta_c^{(k)}$$

thus

$$\hat{\beta}_c^{(k)} = \begin{bmatrix} X'_{J^*} \tilde{X}_0 \\ X'_{J^*} \tilde{X}_1 \\ X'_{J^c} [\tilde{X}_0 + \frac{\sigma_0^2}{\sigma_1^2} \tilde{X}_1] \end{bmatrix}^{-1} \begin{bmatrix} X'_{J^*} \tilde{Y}_0 \\ X'_{J^*} \tilde{Y}_1 \\ X'_{J^c} [\tilde{Y}_0 + \frac{\sigma_0^2}{\sigma_1^2} \tilde{Y}_1] \end{bmatrix} \quad (21)$$

²⁷First transpose each set of condition to obtain two vectors $J^* \times 1$ and a vector $J^c \times 1$ and then rearrange terms.

Now we turn to σ s. For the unconstrained case, $\sigma_0 \neq \sigma_1$, the first-order condition is the same as the one discussed in the section above—equation (16). Note that in this case the first-order conditions of β s are not independent of the first-order conditions for σ_i^2 . Moreover, in this case the model is not linear in these parameters. Thus we proceed as follows.

First we compute (21) using $\sigma_i^{2(k-1)}$ from the previous iteration. Given the estimated $\hat{\beta}_c^{(k)}$ we use (16) to obtain a new set of $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$ and we iterate in this way until convergence.

In the constrained case $\sigma_0 = \sigma_1 = \sigma_c$, $\hat{\beta}_c$ is independent of σ s. The first-order condition with respect σ_c implies²⁸

$$\hat{\sigma}_c^{2(k)} = \frac{\tilde{P}'_0 \xi_0^2 + \tilde{P}'_1 \xi_1^2}{[\tilde{P}_0 + \tilde{P}_1]' \mathbf{1}_{NT}}$$

8.4 SEM Algorithm to compute the covariance matrix

To compute the covariance matrix we follow [?]. The method is based on the information matrix of the complete-data likelihood. Accordingly, consider the information matrix of the complete-data likelihood (9), conveniently partitioned:

$$I_o(\theta) = - \begin{bmatrix} \frac{\partial^2 \log L(\theta)}{\partial \beta \partial \beta} & \frac{\partial^2 \log L(\theta)}{\partial \beta \partial \sigma} & \mathbf{0}_{JW} \\ \frac{\partial^2 \log L(\theta)}{\partial \sigma \partial \beta} & \frac{\partial^2 \log L(\theta)}{\partial \sigma \partial \sigma} & \mathbf{0}_{2W} \\ \mathbf{0}_{WJ} & \mathbf{0}_{W2} & \frac{\partial^2 \log L(\theta)}{\partial \phi \partial \phi} \end{bmatrix}$$

The inverse of I_o gives the large-sample complete-data covariance matrix (see equation 9). Next, replacing the indicator functions by the smoothed probabilities compute the expected complete-data information matrix evaluated at θ^* , i.e. evaluated using the parameter values obtained in the EM procedure:

$$I_{oc} = E[I_o(\theta) | \text{observed data}] \Big|_{\theta=\theta^*}$$

The observed-data covariance matrix, V , can be written as

$$V = I_{oc}^{-1} + I_{oc}^{-1} DM(1 - DM)^{-1}$$

where the second term is the increase in the variance due to the fact that we do not observe the complete data. Let r_{mn} be the mn -th component of DM . To compute r_{mn} let

$$\theta^{(k)}(m) = [\theta_1^*, \dots, \theta_{m-1}^*, \theta_m^k, \theta_{m+1}^*, \dots, \theta_{2J+2+2W}^*]$$

Note that we know θ^* from the EM algorithm. Run once the EM algorithm using $\theta^{(k)}(m)$ and obtain $\theta^{(k+1)}(m)$; then compute the ratios:

$$r_{mn} = \frac{\theta_n^{(k+1)}(m) - \theta_n^*}{\theta_m^{(k)} - \theta_m^*}$$

Finally, for completeness, note that the elements of the expected Hessian matrix of the complete-data likelihood (needed to compute I_{oc}) are as follow. Recall $\beta_c = [\beta_{01} \dots \beta_{0J^*} \beta_{11} \dots \beta_{1J^*} \beta_{c1} \dots \beta_{cJ^c}]'$, let $\sigma_c^2 = [\sigma_0^2 \sigma_1^2]'$ for the unconstrained case and a scalar for the constrained case. Consider $\theta_c = [\beta_c' \phi_0' \phi_1' \sigma_c']'$ and recall: J^* is the number of unconstrained parameters in β_c , J^c is the number of constrained parameters, W is the number of elements in ϕ_i and let $F = 1, 2$ be the number of elements

²⁸Note that $\tilde{P}_0 + \tilde{P}_1$ is a vector of ones, except for the first elements of each line.

in σ_c . Consider the following partition of the Hessian:

$$H = \begin{array}{c|c|c|c|c|c} \hline A_{J^* \times J^*}^1 & \mathbf{0}_{J^* \times J^*} & A_{J^* \times J^c}^2 & \mathbf{0}_{J^* \times W} & \mathbf{0}_{J^* \times W} & D_{J^* \times F}^1 \\ \hline & A_{J^* \times J^*}^3 & A_{J^* \times J^c}^4 & \mathbf{0}_{J^* \times W} & \mathbf{0}_{J^* \times W} & D_{J^* \times F}^2 \\ \hline & & B_{J^c \times J^c} & \mathbf{0}_{J^c \times W} & \mathbf{0}_{J^c \times W} & D_{J^c \times F}^3 \\ \hline & & & C_{W \times W}^1 & \mathbf{0}_{W \times W} & \mathbf{0}_{W \times F} \\ \hline & & & & C_{W \times W}^2 & \mathbf{0}_{W \times F} \\ \hline & & & & & D_{F \times F}^4 \\ \hline \end{array} \quad (22)$$

Let β_{ij^*} be an unconstrained parameter $j^* = 1 \dots J^*$ and let β_{cj^c} be a constrained parameter $j^c = 1 \dots J^c$. For any two unconstrained parameters indexed by j^* and $j^{*'}$, the elements of the partitions A^1 and A^3 , are given by:

$$E_{\Omega_T} \frac{\partial^2 \log L(\theta)}{\partial \beta_{ij^*} \partial \beta_{ij^{*'}}} = - \sum_{\ell} \sum_t \mathbb{P}_{\ell t}(i) x_{\ell t}(j^*) x_{\ell t}(j^{*'}) \frac{1}{\sigma_i^2}.$$

where Ω_T is complete set of information available. Similarly, the elements of the partitions A^2 and A^4 are given by

$$E_{\Omega_T} \frac{\partial^2 \log L(\theta)}{\partial \beta_{ij^*} \partial \beta_{cj^c}} = - \sum_{\ell} \sum_t \mathbb{P}_{\ell t}(i) x_{\ell t}(j^*) x_{\ell t}(j^c) \frac{1}{\sigma_i^2}.$$

Now, consider any two constrained parameters indexed by $j^c, j^{c'}$, the elements of B are given by

$$E_{\Omega_T} \frac{\partial^2 \log L(\theta)}{\partial \beta_{cj^c} \partial \beta_{cj^{c'}}} = - \sum_{\ell} \sum_t \mathbb{P}_{\ell t}(0) x_{\ell t}(j^c) x_{\ell t}(j^{c'}) \frac{1}{\sigma_0^2} + \mathbb{P}_{\ell t}(1) x_{\ell t}(j^c) x_{\ell t}(j^{c'}) \frac{1}{\sigma_1^2}$$

The elements of C^1 and C^2 are given by

$$E_{\Omega_T} \frac{\partial^2 \log L(\theta)}{\partial \phi_{iw} \partial \phi_{iv}} = - \sum_{\ell} \sum_t \mathbb{P}_{\ell t-1}(i) z_{\ell t-1}(w) z_{\ell t-1}(v) p_{\ell t}^{ii} (1 - p_{\ell t}^{ii})$$

Let $\sigma_0 \neq \sigma_1$ then the non-zero elements of D^1 and D^2 are given by

$$E_{\Omega_T} \frac{\partial^2 \log L(\theta)}{\partial \beta_{ij^*} \partial \sigma_i^2} = - \sum_{\ell} \sum_t \mathbb{P}_{\ell t}(i) (y_{\ell t} - x'_{\ell t} \beta_i^c) x_{\ell t}(j^*) \frac{1}{(\sigma_i^2)^2}$$

the elements of D^3 are given by

$$E_{\Omega_T} \frac{\partial^2 \log L(\theta)}{\partial \beta_{cj^c} \partial \sigma_i^2} = - \sum_{\ell} \sum_t \mathbb{P}_{\ell t}(i) (y_{\ell t} - x'_{\ell t} \beta_i^c) x_{\ell t}(j^c) \frac{1}{(\sigma_i^2)^2}$$

and the non-zero elements of D^4 by

$$E_{\Omega_T} \frac{\partial^2 \log L(\theta)}{\partial \sigma_i^2 \partial \sigma_i^2} = - \sum_{\ell} \sum_t \mathbb{P}_{\ell t}(i) \left[(y_{\ell t} - x'_{\ell t} \beta_i^c)^2 \frac{1}{(\sigma_i^2)^3} - \frac{1}{2(\sigma_i^2)^2} \right]$$

Finally, let $\sigma_0 = \sigma_1 = \sigma_c$ then the elements of D^1 and D^2 are given by

$$E_{\Omega_T} \frac{\partial^2 \log L(\theta)}{\partial \beta_{ij^*} \partial \sigma_c^2} = - \sum_{\ell} \sum_t \mathbb{P}_{\ell t}(i)(y_{\ell t} - x'_{\ell t} \beta_i^c) x_{\ell t}(j^*) \frac{1}{(\sigma_c^2)^2}$$

the elements of D^3 are given by

$$\begin{aligned} E_{\Omega_T} \frac{\partial^2 \log L(\theta)}{\partial \beta_{cj^c} \partial \sigma_c^2} &= - \sum_{\ell} \sum_t \mathbb{P}_{\ell t}(0)(y_{\ell t} - x'_{\ell t} \beta_0^c) x_{\ell t}(j^c) \frac{1}{(\sigma_c^2)^2} \\ &\quad + \mathbb{P}_{\ell t}(1)(y_{\ell t} - x'_{\ell t} \beta_1^c) x_{\ell t}(j^c) \frac{1}{(\sigma_c^2)^2} \end{aligned}$$

and the elements of D^4 by

$$\begin{aligned} E_{\Omega_T} \frac{\partial^2 \log L(\theta)}{\partial \sigma_c \partial \sigma_c^2} &= - \sum_{\ell} \sum_t \mathbb{P}_{\ell t}(0) \left[(y_{\ell t} - x'_{\ell t} \beta_0^c)^2 \frac{1}{(\sigma_c^2)^3} - \frac{1}{2(\sigma_c^2)^2} \right] \\ &\quad + \mathbb{P}_{\ell t}(1) \left[(y_{\ell t} - x'_{\ell t} \beta_1^c)^2 \frac{1}{(\sigma_c^2)^3} - \frac{1}{2(\sigma_c^2)^2} \right] \end{aligned}$$

Table 1

Parameter	constant	market concentration	country share	inflation	volatility of exch.rate
β_0^e	0.0487** (0.0170)	0.0410** (0.00001)	0.0675** (0.0214)	0.0738** (0.0189)	0.0428** (0.0146)
β_0^ψ	0.0701** (0.0170)	0.0741** (0.0163)	0.0652** (0.0200)	0.0708** (0.0188)	0.0688** (0.0161)
β_0^{PL}	0.5027** (0.0548)	0.5172** (0.0381)	0.5507** (0.0639)	0.5599** (0.0582)	0.4641** (0.0561)
β_1^e	0.1613** (0.0782)	0.1625** (0.0731)	0.1721* (0.0921)	0.1512** (0.0455)	0.1609** (0.0754)
β_1^ψ	0.0415 (0.1044)	0.0426 (0.1015)	0.0449 (0.1235)	0.0497 (0.1008)	0.0433 (0.1053)
β_1^{PL}	0.9149** (0.3266)	0.9098** (0.3104)	0.9512** (0.3827)	0.9600** (0.2799)	0.9017** (0.3077)
β_1^Y	-0.2843** (0.1233)	-0.2446** (0.0074)	-0.3058** (0.1300)	-0.2052 (0.1273)	-0.2879** (0.1269)
ϕ_{01}	0.5468** (0.2300)	0.1955 (0.1763)	1.2013** (0.3176)	0.3050 (0.3387)	0.1608 (0.3366)
ϕ_{02}		-2.6878** (1.0425)	-0.0185** (0.0079)	0.8127* (0.4281)	0.0211 (0.0178)
ϕ_{03}				0.2571 (0.6918)	
ϕ_{11}	0.4068* (0.2182)	0.0517 (0.1796)	1.0476** (0.3232)	-0.3849 (0.3906)	-0.4860 (0.3604)
ϕ_{12}		-3.9405** (0.7197)	-0.0573** (0.0098)	1.0655** (0.0000)	0.0601** (0.0205)
ϕ_{13}				0.5782 (0.4362)	
σ_0^2	0.0002** (0.0001)	0.0002** (0.0000)	0.0004** (0.0001)	0.0004** (0.0000)	0.0002** (0.0000)
σ_1^2	0.0127** (0.0015)	0.0125** (0.0014)	0.0146** (0.0018)	0.0141** (0.0018)	0.0123** (0.0012)
Avg. $\frac{\partial p^{00}}{\partial Z}$		-0.6284	-0.0039	0.1741	0.0049
Avg. $\frac{\partial p^{11}}{\partial Z}$		-0.8957	-0.0106	0.2519	0.0137
Likelihood	539.4300	545.4033	550.9184	536.9687	547.7699
Obs.	583	583	583	583	583
RMSE	0.0787	0.0787	0.0783	0.0786	0.0788
AIC	-4.8909	-4.8829	-4.8928	-4.8783	-4.8817
LR Test of $\beta_0^e = \beta_1^e$	0.0084	0.0061	0.0044	0.0156	0.0105

Notes:

- All specifications include fixed effects that are constrained to be the same across the two regimes.
- Avg. $\frac{\partial p^{00}}{\partial Z}$ and Avg. $\frac{\partial p^{11}}{\partial Z}$ shows the marginal effect on the conditional probabilities of one percent increase in the corresponding variable in the transition probabilities. In the inflation specification, Z corresponds to the US inflation rate.
- In the last row, the statistics for the likelihood ratio test of the equality of the pass-through coefficients are reported. For a p-value of 0.05, the corresponding chi-square value is 0.00004.

Table 2

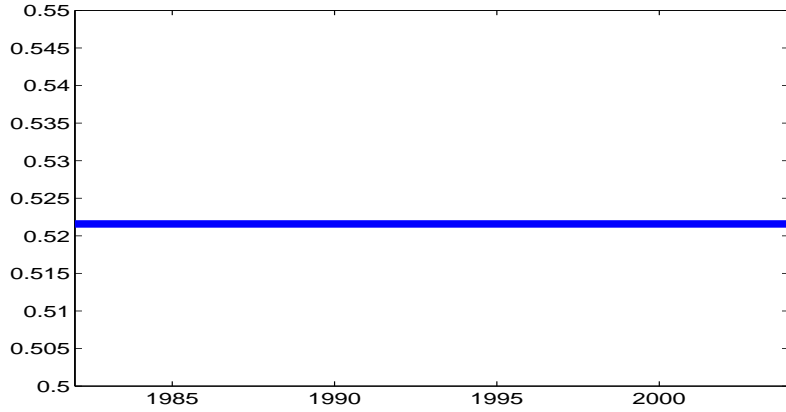
Pricing Equation Parameters		β_0^c	β_0^ψ	β_0^{PL}	β_1^c	β_1^ψ	β_1^{PL}	β_1^Y	σ_0^2	σ_1^2
		0.0572** (0.0000)	0.0568** (0.0172)	0.6350** (0.0000)	0.1557* (0.0854)	0.0650 (0.1081)	0.8867** (0.3463)	-0.2150** (0.0517)	0.0003** (0.0001)	0.0136** (0.0015)
Transition Equation Parameters		constant	market concentration	country share	inflation					
		ϕ_{01}	ϕ_{02}	ϕ_{03}	ϕ_{04}	ϕ_{11}	ϕ_{12}	ϕ_{13}	ϕ_{14}	
		0.6348* (0.3643)	-2.2952** (0.0000)	-0.0214** (0.0062)	0.9048** (0.2883)	0.1072 (0.3089)	-3.6210** (0.0000)	-0.0346** (0.0155)	1.4664** (0.0000)	
			Avg. $\frac{\partial p}{\partial Z}$ -0.4912	Avg. $\frac{\partial p}{\partial Z}$ -0.0046	Avg. $\frac{\partial p}{\partial Z}$ -0.0070				Avg. $\frac{\partial p}{\partial Z}$ 0.1936	Avg. $\frac{\partial p}{\partial Z}$ 0.2983
		Likelihood	RMSE	AIC	LR Test of $\beta_0^c = \beta_1^c$					
		558.0706	0.0788	-4.8638	0.0218					

Notes:

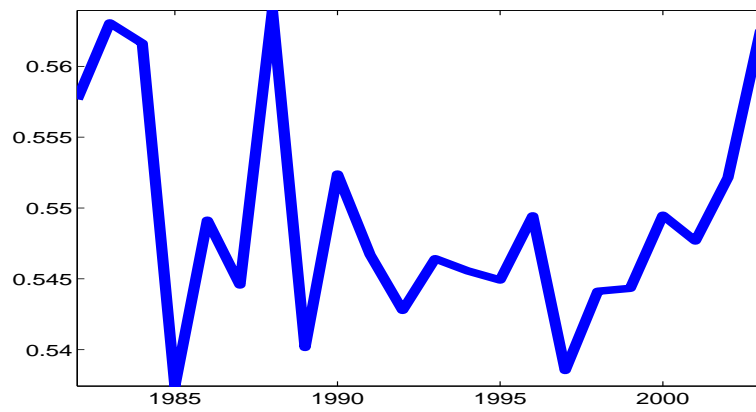
- The specification includes fixed effects that are constrained to be the same across the two regimes.
- Avg. $\frac{\partial p}{\partial Z}$ and Avg. $\frac{\partial p^{11}}{\partial Z}$ shows the marginal effect on the conditional probabilities of one percent increase in the corresponding variable in the transition probabilities. In the inflation specification, Z is the US inflation rate.
- In the last row, the statistics for the likelihood ratio test of the equality of the pass-through coefficients are reported. For a p-value of 0.05, the corresponding chi-square value is 0.00004.

Figure 1: Calculated Fraction of Firms in Low Pass-Through

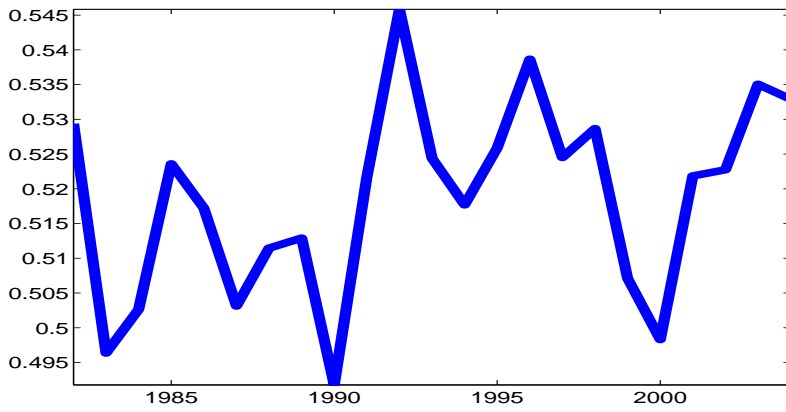
(A) with Constant CPs



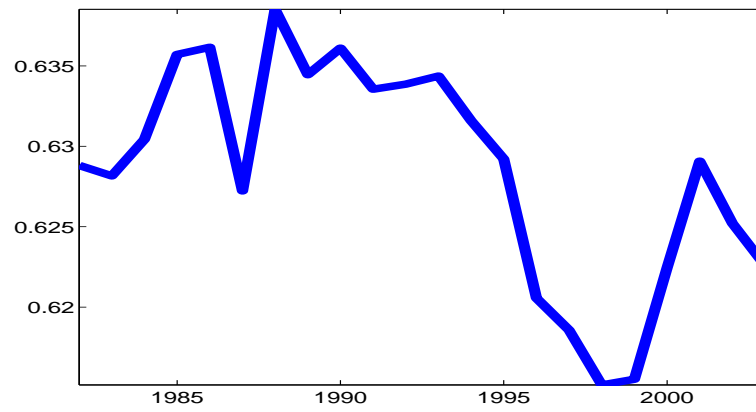
(B) with CPs Driven by Segment Shares



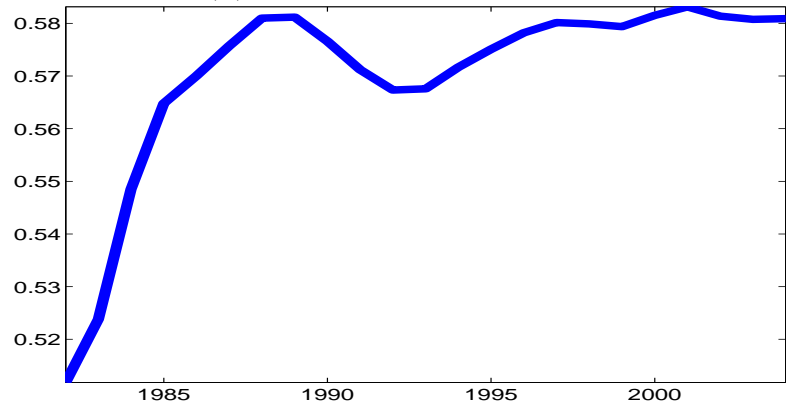
(C) with CPs Driven by Industry Concentration



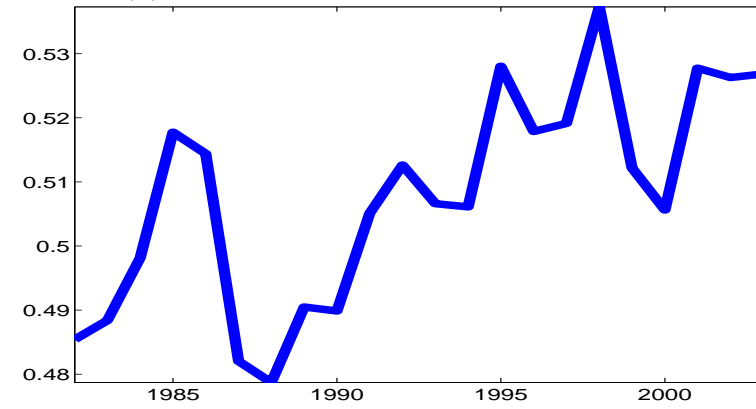
(D) with CPs Driven by Country Shares



(E) with CPs Driven by Inflation



(F) with CPs Driven by Volatility of Exch. Rate



Note: The Fraction of firms in the low pass-through regime is calculated based on the conditional probabilities (CPs) g_{lt}^{00} and g_{lt}^{11} estimated jointly with the pricing equations. The calculation uses equation (7) in the text.

Figure 2A

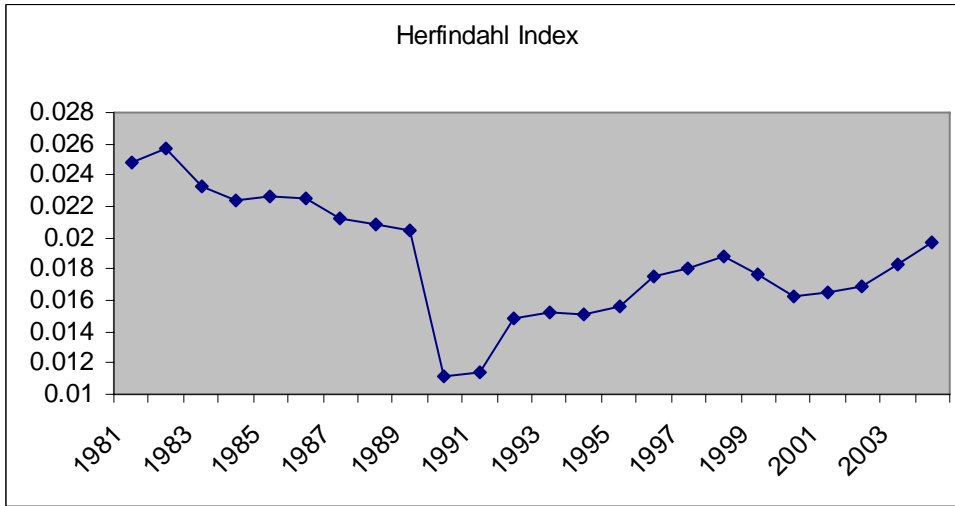


Figure 2B

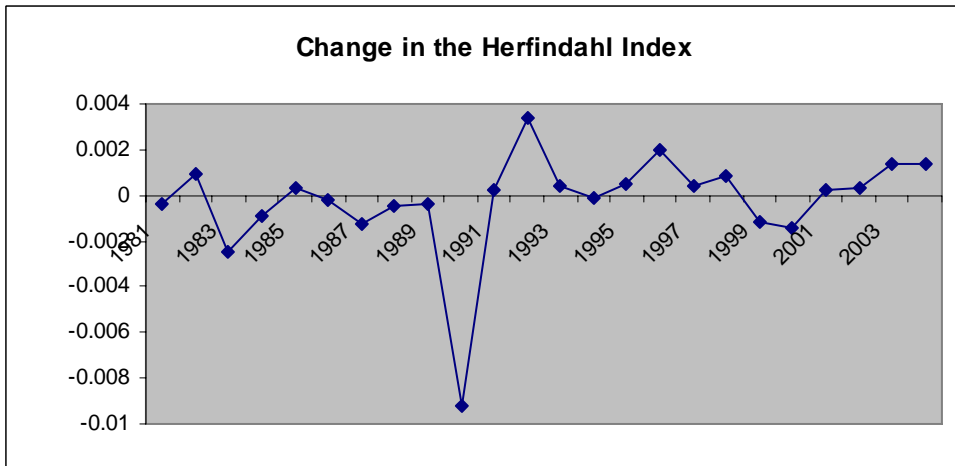


Figure 3: Country Market Shares and Fraction of Firms in the LPT Regime

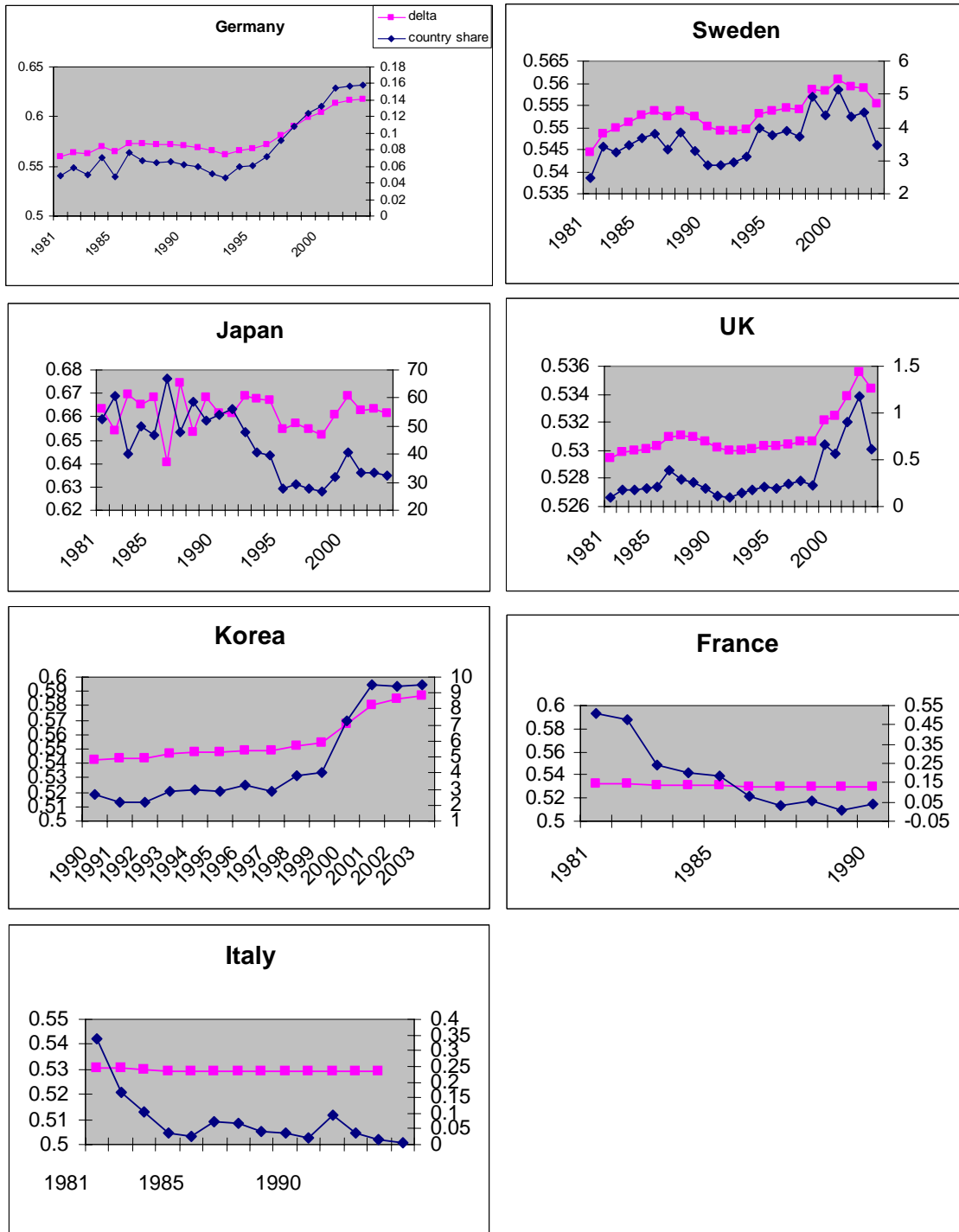


Figure 2: Estimated Exchange Rate Pass-Through Coefficient

