ECG752 - Time Series Econometrics - Spring 2009
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Problem Set 2

Hint for Question 6

A general hint for Question 6 is that we always want to write the elements we study as some combination of $D_T^{-1}X'XD_T^{-1}$ (so we know that at the limit this will be Q) and $D_T^{-1}X'u$ (so we know that at the limit we get a Normal distribution).

Part 1) We know that

$$\hat{\sigma}_T^2 = \frac{1}{T-2}\hat{u}'\hat{u}$$

where $\hat{u} = [\hat{u}_1, \ldots, \hat{u}_T]'$:

- Write $\hat{u}$ as $Y - x\hat{\beta}$ and do the substitution. Remember that $\hat{\beta} = (X'X)^{-1}X'Y$.
- When you get an $X'X$, introduce $D_T D_T^{-1}$ so that $X'X$ becomes $D_T^{-1}X'XD_T^{-1}$.
- We know that $D_T X'u$ goes to something stable, *i*.e. a Normal distribution. What would then happen if you divide this quantity by T, *i.e.* $\frac{1}{T}D_T X'u$? Does it converge to something?

Part 2) The trick for the second part is to study the behavior of the denominator:

- What is $\hat{m}_{1,1}$? It can be written as

$$\hat{m}_{1,1} = \begin{bmatrix} 1 & 0 \end{bmatrix} X'X \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- Make $D_T^{-1}X'XD_T^{-1}$ appear:

$$\hat{m}_{1,1} = \begin{bmatrix} 1 & 0 \end{bmatrix} D_T^{-1}D_T X'X D_T D_T^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- What is $\begin{bmatrix} 1 & 0 \end{bmatrix} D_T^{-1}$? Is there a power of T that you could factorize and move to the numerator, making it stable in the process?
- For τ_δ, it is very much the same expcet that we look at using $\begin{bmatrix} 0 & 1 \end{bmatrix}$ instead of $\begin{bmatrix} 1 & 0 \end{bmatrix}$.

Part 3)

- Under H_0, $a_1\alpha_0 + a_2\delta_0 = r$ and so

$$\begin{aligned} a_1\hat{\alpha}_T + a_2\hat{\hat{\delta}}_T - r &= a_1\hat{\alpha}_T + a_2\hat{\hat{\delta}}_T - (a_1\alpha_0 + a_2\delta_0) \\ &= a_1(\hat{\alpha}_T - \alpha_0) + a_2(\hat{\hat{\delta}}_T - \delta_0) \end{aligned}$$

  So if you multiply by $T^{1/2}$ as asked in the question, you get

$$T^{1/2}(a_1\hat{\alpha}_T + a_2\hat{\hat{\delta}}_T - r) = T^{1/2}[a_1(\hat{\alpha}_T - \alpha_0)] + T^{1/2}[a_2(\hat{\hat{\delta}}_T - \delta_0)]$$

- Is $T^{1/2}$ fast enough for $\hat{\hat{\delta}}_T - \delta_0$?
- When you study the denominator of τ_1, you end up studying $Ta'(X'X)^{-1}a$. You again want to introduce $D_T D_T^{-1}$ so as to get $D_T(X'X)^{-1}D_T$.
- What is the limit of $T^{1/2}a'D_T^{-1}$?

Part 4) More $D_T D_T^{-1}$.