

ECG752 - Time Series Econometrics - Spring 2009
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Computer Exercise 3

The goal of this question is to estimate a GARCH(1,1) model by QMLE for the NASDAQ index. Using the estimated model, you then need to compute over time the one-day 1% Value-at-Risk (VaR) for a portfolio consisting of 1\$ invested in the NASDAQ index. Finally, looking at these VaR's, assess the specification of this GARCH model.

The data series for this question is the daily closing value of the NASDAQ index from February 5, 1971 until April 4, 2007. The data is in the file `nasdaq_daily_clean.csv`. You need to model the variance of the return on the index using a GARCH(1,1). Denote by x_t the value of the index on day t . Denote by y_t the percentage log-return on the index, *i.e.* $y_t = 100(x_t - x_{t-1})$.

Question 1

Estimate by Quasi Maximum Likelihood the following model:

$$y_t = \sigma_t \epsilon_t \tag{1}$$

$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{2}$$

with ϵ_t i.i.d., mean equal to zero and variance equal to one. You need to impose in the estimation the positivity constraints ($\omega > 0$, $\alpha > 0$ and $\beta > 0$) and the stationarity constraint ($\alpha + \beta < 1$).

Because I want you do to an out-of-sample analysis later, you should not use the last 3000 observations when estimating the parameters of the GARCH model.

You must also compute the standard errors of the estimates. Since you are performing QMLE, you need to use the sandwich form for the variance, *i.e.* $I^{-1} J J^{-1}$.

Report the estimates and the standard errors. Comment on the value of the estimates.

Question 2

Using the estimated GARCH model, compute the 1% one-day VaR for the remaining 3000 observations assuming that everyday you have 1\$ invested in the NASDAQ index. Plot this VaR over time and comment.

Since you do not believe that ϵ_t might have a Normal distribution (the tails would be too thin), to compute the 1% VaR you need to use the 1% (empirical) quantile of the standardized residuals $\hat{\epsilon}_t = y_t / \hat{\sigma}_t$.

Question 3

Remember that the Value-at-Risk ($VaR_{t+1|t}$) is an amount computed at time t for which the conditional probability of losing more than this amount at time $t + 1$ is 1%:

$$\Pr(R_{t+1} < VaR_{t+1|t} | I_t) = 0.01. \quad (3)$$

For the last 3000 observations, compare every day the VaR computed for this day with the actual return. From this create the following hit sequence:

$$hit_t = \begin{cases} 1 & \text{if } R_t < VaR_{t|t-1} \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Answer the following questions:

- How many times is hit_t is equal to one?
- If this GARCH model is the correct model for y_t , how many times would you expect to get a one for a sample size of 3000? Briefly explain.
- Assuming that the observations from the hit sequence hit_t are independent, test that the average frequency of hits, $(\sum_t^{3000} hit_t)/3000$, is equal to 0.01. From this test what do you conclude about the specification of this model (does it correctly model the dynamic of y_t)?