1) In the following circuit, A, B and C are circuit elements performing certain function according to the elements identity

![Circuit Diagram]

a) For A = resistor R, B = capacitor C₁, C = capacitor C₂ find the expression of the frequency response of this circuit

The circuit is given by:

![Circuit Diagram with Z₁]

This circuit can be simplified by having the Rand C₁ combined in single equivalent impedance \( Z₁ = \frac{R}{1 + jωRC₁} \), hence

\[
V_o(j\omega) = \frac{1}{Z₁ + jωC₂} \cdot V_i(j\omega) = \frac{1}{1 + jωZ₁C₂} \cdot V_i(j\omega) = \frac{1}{1 + jωRC₂} \cdot V_i(j\omega) = \frac{1 + jωRC₁}{1 + jω(C₁ + C₂)} \cdot V_i(j\omega)
\]

Then the frequency response is

\[
H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1 + jωRC₁}{1 + jωC₁(C₁ + C₂)} = \frac{(1 + jωRC₁)[1 + jωR(C₁ + C₂)]}{1 + ω²R²(C₁ + C₂)}
\]

\[
= \frac{1 - ω²R²(C₁ + C₂) + jωR(2C₁ + C₂)}{1 + ω²R²(C₁ + C₂)}
\]

From which \( Re(H) = \frac{1 - ω²R²(C₁ + C₂)}{1 + ω²R²(C₁ + C₂)} \) and \( Im(H) = \frac{ωR(2C₁ + C₂)}{1 + ω²R²(C₁ + C₂)^2} \)
b) If the element B is removed while A and C still the resistor R and capacitor C₂, respectively, what is the magnitude and the phase of the frequency response and what is the functionality of the circuit? Make a sketch of the magnitude of the frequency response.

Removal of B makes the circuit a RC with the series R and

![Diagram of a RC circuit](image)

\[ V_0(j\omega) = \frac{1}{R + j\omega C_2} V_i(j\omega) = \frac{1}{1 + j\omega RC_2} V_i(j\omega) \]

\[ H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{1 + j\omega RC_2} = \frac{1}{\sqrt{1 + (\omega RC_2)^2}} e^{-j \arctan(\omega RC_2)} \]

Where \( \omega_0^2 = \frac{1}{RC_2} \)

This is a low-pass filter; a sketch of the response has the shape:

![Graph of H vs. Frequency](image)

It also works as an integrator when \( \omega >> 1/RC \)

c) If A is removed from the circuit, B remains =C₁ but element C was replaced by a resistor R₁, find the new functionality of the circuit, i.e. its frequency response and its behavior as a circuit.
Removal of A from the circuit while B remains =C1 and element C = resistor R1, then the circuit appears as follow

\[ V_o = \frac{R_1}{R_1 + 1/j\omega C_1} V_i(j\omega) = \frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1} V_i(j\omega), \text{ hence:} \]

\[ H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{j\omega R_1 C_1}{1 + j\omega R_1 C_1} e^{j90^\circ} = \frac{\omega R_1 C_1}{\sqrt{1 + (\omega R_1 C_1)^2}} e^{j[90^\circ - \arctan(\omega R_1 C_1)]} \]

This is a high-pass filter; a sketch of the response has the shape:

It also works as a differentiator when \( \omega << 1/R_1 C_1 \)

2) A set of sensors were installed on one of the coolant pipes of a nuclear reactor, a thermocouple, two pressure transducer, and a flowmeter. Additionally, a cylindrical ionization chamber was also installed to monitor the radiation in the flowing coolant.

a) The thermal sensor is a K-type capable of measuring temperatures up to 1300°C with the following output voltages

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Output Voltage (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>206</td>
<td>10</td>
</tr>
<tr>
<td>540</td>
<td>22</td>
</tr>
<tr>
<td>815</td>
<td>33</td>
</tr>
<tr>
<td>1100</td>
<td>45</td>
</tr>
<tr>
<td>1300</td>
<td>55</td>
</tr>
</tbody>
</table>
Check the linearity of this K-Type thermocouple and roughly plot its Seebeck coefficient versus temperature.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Output Voltage (mV)</th>
<th>Seebeck Coefficient (mV/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>206</td>
<td>10</td>
<td>0.048544</td>
</tr>
<tr>
<td>540</td>
<td>22</td>
<td>0.040741</td>
</tr>
<tr>
<td>815</td>
<td>33</td>
<td>0.040491</td>
</tr>
<tr>
<td>1100</td>
<td>45</td>
<td>0.040909</td>
</tr>
<tr>
<td>1300</td>
<td>55</td>
<td>0.042308</td>
</tr>
</tbody>
</table>

The thermocouple is near linear **ONLY** in the range 540-1100°C, but nonlinear outside this range.

b) The two pressure transducers are located at the inlet and the outlet of the pipe, where the diameter of the pipe at the inlet is twice the diameter at the outlet. Denote the pressure difference between the output and the input as $\Delta P$. In absence of gravitational effects, find a mathematical expression for the quantity of the flow at the pipe’s outlet in terms of the inlet parameters and the pressure difference $\Delta P$.

Bernoulli’s equation  

$$P_{in} + \frac{1}{2} \rho v_{(in)}^2 = P_{out} + \frac{1}{2} \rho v_{(out)}^2$$

$$\therefore \Delta P = P_{out} - P_{in} = \frac{1}{2} \rho (v_{(in)}^2 - v_{(out)}^2)$$

$$\therefore v_{(out)} = \left(v_{(in)}^2 - 2 \frac{\Delta P}{\rho}\right)^{1/2}$$
c) Obtain the mathematical expression of the voltage pulse due to energy $E_{rad}$ deposited from the coolant fluid into the gas-filled cylindrical ionization chamber length $L$ and an inner and outer electrode radii ‘a’ and ‘b’, respectively. The gas ionization energy $E_{iz}$,

Hint: The capacitance of cylindrical geometry is given by $C_{cylindrical} = \frac{2\pi\varepsilon_0\varepsilon_r}{\ell n(b/a)} L$

The number density of the ion-pairs $n = \frac{E_{rad}}{E_{iz}}$

Hence the charge $Q = nq = q\frac{E_{rad}}{E_{iz}}$

The voltage pulse is determined by $Q/C$ where $C$ is the capacitance of the chamber.

$\therefore$ Voltage pulse $V = \frac{Q}{C} = \frac{q\left(\frac{E_{rad}}{E_{iz}}\right)}{2\pi\varepsilon_0\varepsilon_r \frac{L}{\ell n(b/a)}} = \frac{q}{2\pi\varepsilon_0\varepsilon_r L} \frac{E_{rad}}{E_{iz}} \ell n(b/a)$
3) A PWR operates at a 100% power level of 3400 Mwt. The following information is known for this reactor

**Problem Data**

- **Average Fission Cross Section**: 350 barns
- **Fuel Number Density**: $7.12 \times 10^{20} \text{ cm}^{-3}$
- **Energy Per Fission**: 190 Mev
- **Pellet Diameter**: 0.3225 inches
- **Rod Diameter**: 0.374 inches
- **Average Neutron Flux**: $4.596 \times 10^{13} \text{ n/cm}^2\text{-sec}$
- **Rod Height**: 12 ft
- **Core Flow Rate**: $142 \times 10^6 \text{ lbm/hr}$
- **Cold Leg Temperature**: 556 F
- **Specific Heat**: 1.318 Btu/lbm-F
- **Hot Zero Power Tavg (THZP)**: 560 F
- **Doppler Coefficient**: $-2 \times 10^{-5} \text{ F}^{-1}$
- **Moderature Temperature Coefficient**: $-18 \times 10^{-5} \text{ F}^{-1}$

Programmed: $T_{AVG} = T_{HZP} + 30P_{rel}$, where $P_{rel} = \text{Relative Reactor Power [0} - 1]$

Fuel Temperature: $T_{RX} = T_{AVG} + \zeta(P_{rel})$

Core Average Fuel Temperature Change (for constant moderator temperature): $\Delta T_{RX} = 5^\circ \text{F/\% Rated Power}$

Control Rod Worth: $\rho(z) = 2700 \times 10^{-4} \left( \frac{z-H}{H} \right)$ where $z = 0$ is fully inserted and $H$ is the core height.

a) Determine the hot leg temperature
b) Determine the number of fuel rods
c) Assuming the control rods are in their full out position at 100% power, determine the new rod position if the reactor power is to be reduced to 80%.

Note: 1 watt = 3.41214 Btu/hr
1 Mev/cm$^3$-sec = $1.5477 \times 10^{-8}$ Btu/hr-ft$^3$
1 watt = $6.2421 \times 10^8$ ev/sec

**SOLUTION**

a) $T_{HOT} = T_{COLD} + \frac{\dot{Q}}{mC_p}$

$$T_{HOT} = 556 + \frac{3400 \times 10^6 \times 3.41214}{142 \times 10^6 \times 1.318} = 618 \text{ F}$$

b) $\dot{Q} = G\Sigma_f\phi V_{fuel}$

$$V_{fuel} = \frac{\dot{Q}}{G\Sigma_f\phi}$$
\[
V_{\text{fuel}} = \frac{3400 \times 10^4 \times 6.2421 \times 10^8}{190 \times 10^4 \times 350 \times 7.12 \times 10^{39} \times 4.596 \times 10^{13}} = 9.753 \times 10^6 \text{ cm}^3 = 5.952 \times 10^5 \text{ in}^3
\]

\[
V_{\text{fuel}} = n \frac{\pi}{4} D^2 H \Rightarrow n = \frac{4 \sqrt[3]{V_{\text{fuel}}}}{\pi D^2 H}
\]

\[
n = \frac{4}{\pi} 5.952 \times 10^5 = 0.3225 \times 144 = 50,596
\]

c) \[0 = \Delta \rho_{cr} + \Delta \rho_{mod} + \Delta \rho_{rx}\]

\[-\Delta \rho_{cr} = \Delta \rho_{mod} + \Delta \rho_{rx} = \alpha_{mod} \Delta T_{avg} + \alpha_{rx} \Delta T_{rx}\]

\[\Delta T_{avg} = 30 \times \Delta P_{rel}, \quad \Delta T_{avg} = 30 \times (-0.2), \quad \Delta T_{avg} = -6\]

\[\Delta T_{rx} = \Delta T_{avg} + 5 \times 100 \times (-0.2) = -106\]

\[-2700 \times \left( \frac{z - H}{H} \right) = (-18)(-6) + (-2)(-106) \]

\[-\left( \frac{z - H}{H} \right) = 0.1185 \Rightarrow z = 10.58 \text{ ft}\]
1) The operator sets the new electric load (80%) which sets the reference turbine impulse pressure.

The TCV controller sees an error signal between the measured and reference impulse pressure.

\[ E = P_{imp} - \text{Ref} \cdot P_{imp} \]

\[ E > 0 \quad \text{TCV} \downarrow \]

\[ E < 0 \quad \text{TCV} \uparrow \]

To bring the measured \( P_{imp} \) in line with the reference value.

2) The rod control program is based on two error signals.

\[ E_1 = W_{set} - Q_{set} \]

\[ (W_{set})_{new} \quad (Q_{set})_{new} \]

\[ E_2 = \text{Programmed} \cdot T_{avg} - T_{avg} \]

\[ E = G \cdot E_1 + G_2 \cdot E_2 \]
The measured Pimp sets the programmed Tava.

The measured Pimp also implies the turbine output Wexc.

The average coolant temperature is obtained from the measured hot leg and cold leg temperature.

\[ T_\text{ava} = \frac{T_{\text{hor}} + T_{\text{col}}}{2} \]

For the reduction in power maneuver, the TCV closes to reduce Pimp.

\[ \text{Pimp} \downarrow \Rightarrow \text{Programmed Tava} \downarrow \Rightarrow \text{Wexc} \downarrow \]

\[ \left( \frac{\text{Wexc}}{\text{Wexc}_{\text{ref}}} \right) < \text{Open} \Rightarrow E_1 < 0 \]

\[ \frac{\text{Programmed Tava}}{\text{Tava}_{\text{ref}}} \Rightarrow E_2 < 0 \]

\[ E_1 \& E_2 < 0 \Rightarrow E < 0 \Rightarrow \text{Control rods move in to establish } E = 0 \]
b) The feed control valve controller is based on two error signals, a steam flow/feed flow error

\[ E_1 = \text{Flow} - \text{Steam} \]

and a level error

\[ E_2 = \text{Level}_{\text{set}} - \text{Programmed Level}_{\text{set}} \]

\[ E = G_1 E_1 + G_2 E_2 \]

\[ E > 0 \quad \text{FCV} \downarrow \]
\[ E < 0 \quad \text{FCV} \uparrow \]

Feed flow and steam flow are both directly measured properties. Similarly level is also a directly measured property. The programmed level is a function of Elec.

\[ \text{Elec.} = \frac{\text{Welec}}{(\text{Welec})_{\text{ref}}} \]

Where again Welec is inferred from P(t).
For this maneuver as TCV \downarrow \text{Pipe} \downarrow \\
\text{MSteam} \uparrow \text{Water} \uparrow \text{Elec.Peel} \downarrow = \text{Programmed Level} \uparrow \\
E_1 = m_{\text{Flow}} - m_{\text{Steam}} > 0 \\
E_2 = \text{Level}_{\text{a/o}} - \text{Programmed Level}_{\text{a/o}} > 0 \\
E_1 \land E_2 > 0 \implies E>0 \implies \text{FCV closes to reestablish } E=0
c) Impulse Pressure = Piezoresistive Pressure Sensor

Fiber optic

Thermocouples

Flow = DP transmitter across a restriction (Venturi or orifice)

Level = DP transmitter

Rx Power = Uncompensated ion chambers
Short answers

a- Sketch an OpAm that can be used to add signals such that the output is the sum of all inputs and show the relation between the output and input voltages.

\[ V_o = -\frac{Z_f}{Z_s} V_i = -\left(\frac{Z_f}{R_1} V_{i1} + \frac{Z_f}{R_2} V_{i2} + \frac{Z_f}{R_3} V_{i3}\right) = -R_f \left(\frac{V_{i1}}{R_1} + \frac{V_{i2}}{R_2} + \frac{V_{i3}}{R_3}\right) \]

b- Is fission chamber a gas-filled chamber? What is special about such chamber as a radiation measuring detector?

A fission chamber is a gas-filled detector operates in the ionization region, its interior is lined with fissile material “Special Nuclear Materials” (SNM), U-235 (thermal fission), Pu-239 (thermal fission) and U-238 (fast fission).

c- What is a solid state detector, and how it works?

A solid state detector is a semiconductor in which the depletion layer is sensitive to radiation and hence an electric current flow through upon exposure to radiation.

d- Bernoulli’s equation in steady state flow has no time dependence, what term should be included to include time dependence if the flow is not steady, or transient?

When the flow is transient, i.e. not steady state, the force term \( \rho \frac{\partial v}{\partial t} \) must be added.

e- Explain how a strain gauge can be used as pressure sensor.
In a Wheatstone bridge with the strain gauge occupies one of the bridge links, the change in the output voltage is related to the change in the gauge resistance as a result of the acquired stress due to the pressure. 

\[
V_{\text{output}} = \left( \frac{\frac{R_1}{R_3 + R_4}}{R_5} \right) V_{\text{input}}
\]

f) Why is U-235 the fuel of choice for Light Water Reactors?

It is the only naturally occurring fissile isotope.

g) Why are PWRs referred to as natural load followers and not BWRs?

For a PWR

\[
\dot{W}_{\text{load}} \uparrow \downarrow \Rightarrow TCV \uparrow \downarrow \Rightarrow P_{SG} \downarrow \uparrow \Rightarrow T_{AVG} \downarrow \uparrow \Rightarrow \rho \uparrow \downarrow \Rightarrow \dot{Q}_{EX} \uparrow \downarrow
\]

For a BWR

\[
\dot{W}_{\text{load}} \uparrow \downarrow \Rightarrow TCV \uparrow \downarrow \Rightarrow P_{SG} \downarrow \uparrow \Rightarrow \alpha_g \uparrow \downarrow \Rightarrow \rho \downarrow \uparrow \Rightarrow \dot{Q}_{EX} \downarrow \uparrow
\]

h) How does the Pressurizer control pressure in PWRs?

If \( P < P_{\text{SETPOINT}} \)

Sprays \( \downarrow \)  Heaters \( \uparrow \)

If \( P > P_{\text{SETPOINT}} \)

Sprays \( \uparrow \)  Heaters \( \downarrow \)

i) Why is a stability analysis not usually performed for PWRs?

PWRs are designed with negative fuel and moderator temperature coefficients which guarantee stability.

j) What parameters determine a reactors stable period following a reactivity insertion?
If the reactor is not prompt critical: Reactivity, Delayed Neutron Fraction and delayed neutron decay constant.

If the reactor is prompt critical: Reactivity, Delayed Neutron Fraction and Prompt Neutron Lifetime.