1) (40%) The piping system shown below is equipped with pressure and temperature measuring sensors. Temperature is measured by thermocouples and pressure is measured by strain gauges.

![Diagram of piping system](image)

a) If the pressure measured by $P_2$ is half of that measured by $P_1$, derive the equation that relates the velocity in the second section ($v_{z2}$) to the inlet velocity ($v_{z1}$). **Do not neglect the gravitational effect** (known as the potential energy per unit volume).

**SOLUTION**

**Bernoulli’s equation:**

\[
\left( P + \frac{1}{2} \rho v_{z1}^2 + \rho gh \right)_{1st \ section} = \left( P + \frac{1}{2} \rho v_{z2}^2 + \rho gh \right)_{2nd \ section} = \text{constant}
\]

\[
P_1 + \frac{1}{2} \rho v_{z1}^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho v_{z2}^2 + \rho gh_2
\]

\[
\therefore \frac{1}{2} \rho v_{z2}^2 = (P_1 - P_2) + \rho g (h_1 - h_2) + \frac{1}{2} \rho v_{z1}^2
\]

*from which*

\[
v_{z2} = \left( \frac{P_2}{\rho} + 2g(h_1 - h_2) + \frac{v_{z1}^2}{2} \right)^{1/2}
\]

b) What is the quantity (Q) of the liquid flowing into the 2nd section?

**SOLUTION**

\[Q = v_A\]
\[ Q_{2nd\ section} = (vA)_{2nd\ section} = v_2A_2 = \left( \frac{P_2}{\rho} + 2g(h_1 - h_2) + v_{z1}^2 \right)^{1/2} \frac{\pi D_2^2}{4} \]

c) Denoting the viscosity of the fluid as \( \mu_{\text{fluid}} \), write down the equation of the Reynold’s number for this fluid as the fluid flows through the 2\textsuperscript{nd} section

\[ R_e = \frac{\rho v_{z2} D_2}{\mu_{\text{fluid}}} = \frac{\rho D_2}{\mu_{\text{fluid}}} \left( \frac{P_2}{\rho} + 2g(h_1 - h_2) + v_{z1}^2 \right)^{1/2} \]

d) As the pressure sensors are strain gauges, and as the pressure sensor \( P_2 \) measured half of that measured by \( P_1 \), obtain the value of the resistance of gauge 2 (\( R_{g2} \)) with respect to that of gauge 1 (\( R_{g1} \)). The batteries supplying the input voltage for the sensors are identical. Resistors in the bridges (in k\( \Omega \)) are \( R_1=2, R_2=6, R_3=5, R_4=8, R_5=4, R_6=8 \) k\( \Omega \).

\[ V_1 = \left( \frac{R_1}{R_3 + R_{g1}} - \frac{R_2}{R_1 + R_2} \right) V_{\text{input}} \]

\[ V_2 = \left( \frac{R_6}{R_6 + R_{g2}} - \frac{R_5}{R_4 + R_5} \right) V_{\text{input}} \]
\[
\begin{align*}
\frac{V_2}{V_1} &= \frac{P_2}{P_1} = \frac{1}{2} = \frac{R_6 - R_5}{R_6 + R_5} \\
&= \frac{8}{8 + R_{g2}} - \frac{4}{8 + R_{g1}} \\
&= \frac{8}{8 + R_{g2}} - \frac{4}{8 + R_{g1}} \\
&= \frac{8}{8 + R_2} - \frac{1}{5 + R_{g1}} \\
&= \frac{16}{8 + R_{g2}} - \frac{2}{5 + R_{g1}} \\
\therefore \quad \frac{16}{8 + R_{g2}} &= \frac{5}{5 + R_{g1}} \\
&= 40 + R_{g2} = 80 + R_{g1} \\
&= R_{g2} = 40 + R_{g1}
\end{align*}
\]

e) A C-type thermocouple, which has a measuring range up to 2250 °C measures the temperature in the 3rd section. The relation between the temperature and the measured voltage across the thermocouple leads has the form: 

\[ T = 27 + 43 V + 0.5 V^2 \].

Determine the Seebeck coefficient of this thermocouple when the measured voltage is 10mV.

**SOLUTION**

The Seebeck voltage is given by 

\[ \Delta V_{\text{th}} = \alpha \Delta T \],

hence 

\[ V = \alpha \Delta T = 27 + 43 V + 0.5 V^2 \]

\[ 10 = \alpha \ (27 + 43 \times 10 + 0.5 \times 100) \]

\[ \therefore \alpha = \frac{10}{507} = 19.724 \times 10^{-3} \text{ mV / °C} \]

2) (40%)

The one delayed group form of the reactor kinetics equations is

\[ \frac{dn}{dt} = \frac{(\rho - \beta)}{\Lambda} n + \Lambda C \]

\[ \frac{dC}{dt} = \frac{\beta n}{\Lambda} - \Lambda C \]

a) For \( \rho < \beta \), show that shortly after a reactivity insertion, the reactor power increases according to the Prompt Jump Approximation

\[ \frac{dn}{dt} = -\frac{\rho \lambda}{\rho - \beta} n \]

**SOLUTION**

Shortly after the reactivity insertion, the solution for the neutron number density can be approximated by the solution of
i) $0 = \frac{(\rho - \beta)}{\Lambda} n + \lambda C$

ii) $\frac{dC}{dt} = \frac{\beta n}{\Lambda} - \lambda C$

Rewrite i) as

$\frac{(\rho - \beta)}{\Lambda} n = -\lambda C$

and take the time derivative

$\frac{(\rho - \beta)}{\Lambda} \frac{dn}{dt} = -\lambda \frac{dC}{dt}$

from ii)

$\frac{dC}{dt} = \frac{\beta n}{\Lambda} - \lambda C$

$= \frac{\beta n}{\Lambda} + \frac{(\rho - \beta)}{\Lambda} n$

$= \frac{\rho n}{\Lambda}$

$\therefore \frac{(\rho - \beta)}{\Lambda} \frac{dn}{dt} = -\lambda \frac{dC}{dt} = -\lambda \frac{\rho n}{\Lambda}$

$\Rightarrow \frac{dn}{dt} = -\lambda \frac{\rho n}{(\rho - \beta)}$

b) If the power increase follows a stable period of 81.25 seconds, determine the reactivity insertion

**SOLUTION**

From the solution to part a), the period is

$\tau = \frac{(\beta - \rho)}{\lambda \rho}$

Solving for the reactivity

$\rho = \frac{\beta}{\frac{1}{\lambda} + \tau}^{-1}$

For the given data
\[ \rho = \beta \left[ \frac{1}{\lambda} + \tau \right]^{-1} = 0.0075 \left[ \frac{1}{0.08} + 81.25 \right]^{-1} = 0.001 \]

c) Assuming the Prompt Jump Approximation is valid, and

\[ \rho = \alpha_{\text{Qx}} (n - n_0) \]

Show that a negative power coefficient is required for stability.

**SOLUTION**

**Short Solution**

For

\[ \frac{dn}{dt} = -\lambda \frac{\rho n}{(\rho - \beta)} = \lambda \frac{\rho n}{(\beta - \rho)} \]

and \( \rho = \alpha_{\text{Qx}} (n - n_0) < \beta \)

\[ \frac{dn}{dt} = \lambda \frac{\alpha_{\text{Qx}} (n - n_0) n}{\beta - \alpha_{\text{Qx}} (n - n_0)} \]

For any perturbation in \( n \) such that \( n - n_0 = \delta n > 0 \)

\[ \frac{dn}{dt} = \lambda \frac{\alpha_{\text{Qx}} \delta n}{\beta - \alpha_{\text{Qx}} \delta n} n \]

For \( \alpha_{\text{Qx}} (+) \) and \( \alpha_{\text{Qx}} \delta n < \beta \)

\[ \lambda \frac{\alpha_{\text{Qx}} \delta n}{\beta - \alpha_{\text{Qx}} \delta n} > 0 \]

which implies

\[ \frac{dn}{dt} > 0 \quad \text{and power increases without bound, such that the system is unstable.} \]

**For \( \alpha_{\text{Qx}} (-) \)**

\[ \lambda \frac{\alpha_{\text{Qx}} \delta n}{\beta - \alpha_{\text{Qx}} \delta n} < 0 \]

which implies
\[
\frac{dn}{dt} < 0 \quad \text{such that the system is unstable.}
\]

**Longer Solution**

Let \( F(n, \rho) = -\lambda \frac{\rho n}{\rho - \beta} = -\lambda \rho n (\rho - \beta)^{-1} \)

Linearize about fixed reference values \( n_0, \rho_0 \)

\[
F(n, \rho) \approx F(n_0, \rho_0) + (n-n_0) \frac{\partial F}{\partial n}_{n_0, \rho_0} + (\rho-\rho_0) \frac{\partial F}{\partial \rho}_{n_0, \rho_0}
\]

\[
\frac{\partial F}{\partial n}_{n_0, \rho_0} = -\rho_0 \lambda \\
\frac{\partial F}{\partial \rho}_{n_0, \rho_0} = -\lambda n_0 \left[ 1 - \frac{\rho_0}{\rho_0 - \beta} \right]
\]

If the linearization is about an initially critical, steady state solution, then \( \rho_0 = 0 \) and

\[
F(n_0, \rho_0) = 0
\]

\[
\frac{\partial F}{\partial n}_{n_0, \rho_0} = 0 \\
\frac{\partial F}{\partial \rho}_{n_0, \rho_0} = \frac{\lambda n_0}{\beta}
\]

such that

\[
F(n, \rho) \approx \rho \frac{\lambda n_0}{\beta}
\]

and

\[
\frac{dn}{dt} = \rho \frac{\lambda n_0}{\beta} \quad \text{where} \quad \rho = \alpha_{\text{ex}} (n-n_0) < \beta
\]

For any perturbation in \( n \) such that \( n-n_0 = \delta n > 0 \)

\[
\frac{dn}{dt} = \alpha_{\text{ex}} \delta n \frac{\lambda n_0}{\beta}
\]

the right hand side of the equation is (-) only for \( \alpha_{\text{ex}} < 0 \).
Problem Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prompt Neutron Lifetime</td>
<td>$10^{-4}$ s</td>
</tr>
<tr>
<td>Delayed Neutron Fraction</td>
<td>0.0075</td>
</tr>
<tr>
<td>Delayed Neutron Decay Constant</td>
<td>0.08 s$^{-1}$</td>
</tr>
</tbody>
</table>

3) (20%) Short Answers

a) Why do thermocouples vary in their range of temperature measurements?
   
   ANSWER: Because they are made of different metals and hence they have different Seebeck coefficients

b) Why may thermocouples need to be connected to Op Amps?
   
   ANSWER: Because the output voltage can be small and must be amplified

c) What is a ‘fiber optic pressure sensor’?
   
   ANSWER: It is a sensor that a diaphragm that deflects under external applied force causing light intensity to change, thus measures the force/pressure

d) Where should we measure the flow in a PWR nuclear reactor?
   
   ANSWER: In all piping circulating coolants, steam, at the inlets and exits of the pumps

e) What is a ‘variable area flowmeter’?
   
   ANSWER: It is a meter that measures the change in the height of a float, which is directly proportional to the flow rate $d = Q(\Delta t / A)$

f) What is meant by prompt critical and what challenges does it present for reactor control?
   
   ANSWER: The reactor is critical on prompt neutrons alone, with a period on the order of fractions of a second. This implies any control system would have to initial control actions in this same time frame.

g) What is the basis of the 2200 F peak clad temperature limit?
   
   ANSWER: This is the threshold temperature for the exothermic metal-water reaction between Zirconium cladding and water

h) How is a negative fuel temperature coefficient ensured in UO$_2$ fuels?
   
   ANSWER: The negative fuel temperature coefficient is the result of increased parasitic absorption of neutrons in U-238 at elevated temperatures. By utilizing low enrichment (low U-235 content relative to U-238) U in UO$_2$ fuels, a negative fuel temperature coefficient is ensured.