A PWR is at Hot Zero Power (HZP) conditions with control rods full in, i.e. the system is at a temperature corresponding to (Rx Pwr)_{REL} = 0 (557 F). At these conditions, the reactor has characteristics given below. The Doppler only power coefficient is a surrogate for the fuel temperature coefficient, simply expressed in terms of power as opposed to fuel temperature. The control rods are in four banks with their total worth given below and insert reactivity linearly with position as they move out, i.e. at “full out” they insert zero reactivity and at “full in” they insert their full worth. The control rods move in steps, with a 100 step overlap and 228 steps corresponding to full out. 100 step overlap implies that once a bank has moved 128 steps, the next bank begins to move. You can assume the banks move out in order A, B, C, D. Assuming the moderator temperature as a function of reactor power behaves as

\[ T_{mod} = 557 + 33 \times (Rx \ Pwr)_{REL} \quad (Rx \ Pwr)_{REL} \in [0,1] \]


a) Determine the critical rod positions at zero power.
b) The operator begins withdrawing rods to reach full power conditions and realizes full power will not be attainable with the rods in their full out position. What will be the power level (relative) at the full out position?

c) In order to attain full power, the operator reduces the boron concentration in the primary system. What will be the critical boron concentration at full power with rods full out?

**Problem Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core Excess Reactivity</td>
<td>9710 pcm</td>
</tr>
<tr>
<td>Soluble Boron Concentration</td>
<td>1300 ppm</td>
</tr>
<tr>
<td>Moderator Temperature Coefficient</td>
<td>-11.8 pcm/F</td>
</tr>
<tr>
<td>Doppler Only Power Coefficient</td>
<td>-9.8 pcm/% Power</td>
</tr>
<tr>
<td>Soluble Boron Coefficient</td>
<td>-7 pcm/ppm</td>
</tr>
<tr>
<td>Control Bank A Worth</td>
<td>582 pcm</td>
</tr>
<tr>
<td>Control Bank B Worth</td>
<td>973 pcm</td>
</tr>
<tr>
<td>Control Bank C Worth</td>
<td>1171 pcm</td>
</tr>
<tr>
<td>Control Bank D Worth</td>
<td>893 pcm</td>
</tr>
</tbody>
</table>

Note: 1 pcm = 10^{-5} \Delta \rho

**Solution**

a) The reactivity work of the inserted rods at their critical location is

\[ \theta = \rho_b + \rho_B + \rho_{RODS} \]
\[ = 9710 - (7)(1300) + \rho_{RODS} \]

\[ \rho_{RODS} = -610 \text{ pcm} \]

This implies Banks A and B must be full out. Since there is a 100 step overlap, if Bank C is full out, Bank D will be 128 inserted (100 steps withdrawn). The reactivity worth of Bank D at this position is

\[ \rho_D = -893 \left( \frac{128}{228} \right) = -501 \text{ pcm} \]

which implies the critical rod position includes contributions from Banks C and D.
\[ \rho_{\text{rods}} = -610 \text{ pcm} = \rho_c + \rho_d \]

Since the rods insert reactivity linearly with position

\[ \rho_c = -1171 \times \left\{1 - \frac{z_c}{228}\right\} \]

\[ \rho_d = -893 \times \left\{1 - \frac{z_d}{228}\right\} \]

where \( z \) is the number of steps withdrawn. For a 100 step overlap

\[ z_c = 128 + z_d \quad z_d \in [0, 100] \]

\[ -610 = -1171 \times \left\{1 - \frac{128 + z_d}{228}\right\} - 893 \times \left\{1 - \frac{z_d}{228}\right\} \]

\[ z_d = \frac{(1171 + 893 - 610)(228) - (1171)(128)}{1171 + 893} = 88 \text{ steps withdrawn} \]

\[ z_c = 128 + 88 = 216 \text{ steps withdrawn} \]

b) With the rods in their full out position, moderator and fuel temperature feedback compensate for the difference in the excess reactivity and the reactivity due to boron

\[ 0 = \rho_0 + \rho_B + \alpha_{\text{mod}} \Delta T_{\text{mod}} + \alpha_{\text{rx}} \Delta (\text{Rx Pwr})_{\text{REL}} \times 100 \]

\[ = 610 - 11.8 \times (33) \times (\text{Rx Pwr})_{\text{REL}} - 9.8 \times 100 \times (\text{Rx Pwr})_{\text{REL}} \]

\[ (\text{Rx Pwr})_{\text{REL}} = \frac{-610}{-(11.8 \times 33 + 9.8 \times 100)} = 0.445 = 44.5 \% \]

c) To achieve full power, the boron concentration has to be reduced from its initial value to compensate for the higher fuel and moderator temperatures.

\[ 0 = \rho_0 + \rho_B + \alpha_{\text{mod}} \Delta T_{\text{mod}} + \alpha_{\text{rx}} \Delta (\text{Rx Pwr})_{\text{REL}} \times 100 \]

\[ = 9710 - 7 \times C_B - 11.8 \times (33) - 9.8 \times 100 \]

\[ C_B = \frac{9710 - 11.8 \times 33 - 9.8 \times 100}{7} = 1191.5 \text{ ppm} \]