Example

$Z_1 = 1k\Omega$

$R = 1k\Omega$

$R = 10k\Omega$

$Load = Z_L = 10k\Omega$

$Z_2 = 1/j\omega C$

$V_T = V_S \frac{Z_2}{Z_1 + Z_2}$

$V_L = V_T \frac{Z_L}{Z_L + Z_T} = \frac{Z_L}{Z_L + \frac{Z_1 Z_2}{Z_1 + Z_2}} \frac{Z_2}{Z_1 + Z_2}$

$V_S = \frac{Z_L Z_2}{Z_L (Z_1 + Z_2) + Z_1 Z_2} V_S$

\[ \frac{V_L}{V_S}(j\omega) = H_V(j\omega) = \frac{Z_L Z_2}{Z_L (Z_1 + Z_2) + Z_1 Z_2} = \frac{100}{j\omega + 110} \]

OR  \[ H_V(j\omega) = \frac{100}{\sqrt{\omega^2 + (110)^2}} \ e^{j \arctan(\omega/110)} \]

OR  \[ H_V(j\omega) = \frac{100}{\sqrt{\omega^2 + (110)^2}} \angle -\arctan \left( \frac{\omega}{110} \right) \]

While we defined the frequency response by the ratio between the load-to-source voltages as:

$$H_V(j\omega) = \frac{V_L}{V_S}(j\omega)$$

However, we also can defined the frequency response by the ratio between the load-to-source currents as:

$$H_I(j\omega) = \frac{I_L}{I_S}(j\omega)$$

Or the impedance frequency response by ratio between the load voltage-to-the source current as:

$$H_Z(j\omega) = \frac{V_L}{I_S}(j\omega)$$

Or the admittance frequency response by ratio between the load current -to-the source voltage as:

$$H_Y(j\omega) = \frac{I_L}{V_S}(j\omega)$$
Example

We recognize that $I_L$ related to $I_S$ by a current divider and $V_L = R_L I_L$

$$H_Z(j\omega) = \frac{V_L}{I_S}(j\omega)$$

$$I_L(j\omega) = \frac{R_S}{R_S + R_L + j\omega L} I_S(j\omega) = \frac{1000}{1000 + 4000 + j\omega(2\times10^{-3})} I_S(j\omega)$$

$$V_L(j\omega) = R_L I_L(j\omega) = 4000 \times \frac{1000}{5000 + j\omega(2\times10^{-3})} I_S(j\omega)$$

\[ V_L(j\omega) = \frac{4 \times 10^6}{5 \times 10^3 + j\omega \left(2 \times 10^{-3}\right)} I_s(j\omega) \]

\[ \therefore H_Z(j\omega) = \frac{V_L(j\omega)}{I_s(j\omega)} = \frac{4 \times 10^6}{5 \times 10^3 + j\omega \left(2 \times 10^{-3}\right)} \]

\[ \therefore H_Z(j\omega) = \frac{4 \times 10^6}{\sqrt{25 \times 10^6 + 4 \times 10^{-6} \omega^2}} e^{-j \arctan \frac{2 \times 10^{-3} \omega}{5 \times 10^3}} \]

\[ H_Z(j\omega) = \frac{800}{\sqrt{1 + 16 \times 10^{-14} \omega^2}} e^{-j \arctan \left(4 \times 10^{-7} \omega\right)} \]
Let Us Discuss Filters

Simple RC low pass filter

\[ V_o(j\omega) = \frac{1}{j\omega C} \left( \frac{1}{R + 1/j\omega C} \right) \]
\[ V_i(j\omega) = \frac{1}{1 + j\omega RC} V_i(j\omega) \]

\[ H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{1}{1 + j\omega RC} = \frac{1}{\sqrt{1 + (\omega RC)^2}} e^{-j \arctan(\omega RC)} \]

\[ H(j\omega) = \frac{1}{\sqrt{1+(\omega RC)^2}} e^{-j \arctan(\omega RC)} \]

we can write \( H(j\omega) = \left| H(j\omega) \right| e^{j\phi_H(j\omega)} \)

where \( \left| H(j\omega) \right| = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{1+(\omega/\omega_o)^2}} \)

and \( \omega_o^2 = \frac{1}{RC} \) is the cut-off frequency

and \( \phi_H(j\omega) = -\arctan(\omega RC) = -\arctan(\omega/\omega_o) \)

So what is the meaning of this?
It means that this circuit will pass signals at low frequency with \( \omega \ll 1/RC \) and NOT passing signals with frequency \( \omega \gg 1/RC \), i.e. it is filtering out high frequency signal.

\[
V_o = |H|V_i \quad \text{and} \quad \phi_o = \phi_H + \phi_i
\]

Case of RC provides \( \omega_o = 2\pi f \), \( f = 1000\text{Hz} \), observe cut off at 1000
In fact, the circuit also acts as an **integrator** when $\omega \gg 1/RC$ so the output voltage is the integral of the input voltage, i.e. the RC value must be much greater than the angular frequency of the input signal.

\[
V_o = \frac{1}{C} \int idt \quad \text{but} \quad i \approx \frac{V_i}{R} \quad \text{for} \quad \omega \gg 1/RC
\]

i.e. the capacitive impedance is much greater than the series resistor

\[
\therefore V_o \approx \frac{1}{RC} \int V_i dt
\]