Pressure and Flow Sensors and Measurements
The equation of motion of a fluid in a smooth pipe can be written as

$$\rho \frac{d\vec{v}}{dt} = -\nabla P$$

and can be written in the convective derivative as:

$$\rho \left( \frac{d\vec{v}}{dt} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla P$$

in 1-D axial

$$\rho \left( \frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial P}{\partial z}$$
where the acceleration term is written in the convective derivative form

\[
\rho \left( \frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z} \right) = \rho \frac{\partial v_z}{\partial t} + \frac{\partial \left( \rho v_z^2 / 2 \right)}{\partial z}
\]

The equation of motion is (momentum equation) is then:

\[
\rho \frac{\partial v_z}{\partial t} + \frac{\partial}{\partial z} \left( \frac{1}{2} \rho v_z^2 \right) = -\frac{\partial P}{\partial z}
\]

to account for viscous drag we add a loss term \(- \frac{2\tau_w}{R}\) to the RHS of the equation

\[
\rho \frac{\partial v_z}{\partial t} + \frac{\partial}{\partial z} \left( \frac{1}{2} \rho v_z^2 \right) = -\frac{\partial P}{\partial z} - \frac{2\tau_w}{R}
\]
Hence, the macroscopic force equation is

\[ \rho \frac{\partial v_z}{\partial t} + \frac{\partial}{\partial z} \left( \frac{1}{2} \rho v_z^2 \right) = -\frac{\partial P}{\partial z} - \frac{2\tau_w}{R} \]

\[ \rho \frac{\partial v_z}{\partial t} = -\frac{\partial}{\partial z} \left( \rho \frac{1}{2} v_z^2 \right) \]

change in the velocity due to the kinetic energy gradient
\[ \rho \frac{1}{2} v_z^2 \] is also called "the dynamic pressure"

change in velocity due to the axial pressure gradient
\[ \frac{\partial P}{\partial z} \]

velocity loss due to viscous drag along the wall, where the factor \( \frac{2}{R} \) is due to the transfer from surface to volume \( 2\pi R \ell / \pi R^2 \ell \)
In absence of viscous drag the equation goes back to the form

$$\rho \frac{\partial v_z}{\partial t} + \frac{\partial}{\partial z} \left( \frac{1}{2} \rho v_z^2 \right) = -\frac{\partial P}{\partial z} \quad \text{OR} \quad \rho \frac{\partial v_z}{\partial t} + \frac{\partial}{\partial z} \left( P + \frac{1}{2} \rho v_z^2 \right) = 0$$

The steady state condition removes the acceleration and hence

$$\frac{\partial}{\partial z} \left( P + \frac{1}{2} \rho v_z^2 \right) = 0$$

And upon integration, one can clearly obtain the condition

$$P + \frac{1}{2} \rho v_z^2 = \text{constant}$$

This is the Bernoulli’s equation with no gravitational effect

$$P + \frac{1}{2} \rho v_z^2 + \rho gh = \text{constant}$$
\[ \rho \frac{\partial v_z}{\partial t} + \frac{\partial}{\partial z} \left( P + \frac{1}{2} \rho v_z^2 \right) + \frac{2 \tau_w}{R} = 0 \]

The viscous drag term can be written as \( \tau_w = \frac{1}{2} \rho v_z^2 C_F \)

\[ \rho \frac{\partial v_z}{\partial t} + \frac{\partial}{\partial z} \left( P + \frac{1}{2} \rho v_z^2 \right) + \frac{1}{2R} \rho v_z^2 C_F = 0 \]

And \( C_F \) is the friction coefficient and depends on the flow regime

It appears that a measure of the pressure, and differential pressure, also determines the nature of the flow and the floe rate

Let us see how can we measure the pressure

With no acceleration term and using the Bernoulli’s equation with the gravitational term neglected, then;

\[ \left( P + \frac{1}{2} \rho v_z^2 \right)_{\text{in}} = \left( P + \frac{1}{2} \rho v_z^2 \right)_{\text{out}} \]
Bernoulli Equation: conservation of energy principle appropriate for flowing fluids.

Consider pressure to be energy density.

In high velocity flow through the constriction, kinetic energy must increase at the expense of pressure energy.
Omegadyne DPG409 Series Digital Pressure Gauge (DPG) with Analog Output and Optional Wireless Transmitter

Omega PX09GW Series Submersible Pressure Transmitters

Omega High Accuracy Voltage Output Pressure Transducer with Shunt Calibrator for Quick Calibration Checks

PX951-5KS5V is a 0 to 5,000 psi Sealed Gage pressure transducer

Kistler Type 4264A Piezoresistive Pressure Transmitter (differential) Pressure Range 350 bar
Kistler Type 4045A200 Piezoresistive Absolute Pressure Sensor up to 200 bar

Type 4603B Piezoresistive Amplifier Process Controlled
Luna Inc. Nuclear Reactor Pressure Sensor
High temperature fiber optic pressure sensor that can operate in the harsh environment of a nuclear reactor core

Discussion: Where to measure pressures and flow?
PWR
Red circles added by instructor to show possible locations for pressure and flow monitoring

http://www.nrc.gov/reading-rm/basic-ref/students/animated-pwr.html
BWR

Discuss:
possible
locations for
pressure and
flow monitoring

Omega
Pre-wired strain gauge

Dimensions Key:
GRID
A: Active gage length
B: Active gage width
CARRIER
C: Matrix length
D: Matrix width
\[ \text{Stress} = \sigma = \frac{F}{A} \]
\[ \text{Strain} = \varepsilon = \frac{\Delta L}{L} \]
\[ \text{Gauge Factor} \ GF = \left( \frac{\Delta R}{R} \right) / \left( \frac{\Delta L}{L} \right) = \frac{1}{\varepsilon} \left( \frac{\Delta R}{R} \right) \]

\[ V_{output} = V_{CD} - V_{CB} \]
\[ V_{output} = \left( \frac{R_3}{R_3 + R_g} - \frac{R_2}{R_1 + R_2} \right) V_{input} \]

Wheatstone bridge

\[ \text{Strain} = \varepsilon = \frac{\Delta R_g / R_g}{GF} \]
Gauge-Factor (GF) Temperature Dependence
The diaphragm deflects when an external load is applied on it. The variation of the height of the Fabry-Perot cavity below the diaphragm causes the intensity of the light incident on the optical fiber to change thereby producing the desired transduction result.

\[
\Delta L = \frac{3\left(1-\nu^2\right)a^4}{16Ed^3}
\]

where \(\nu\) and \(E\) are the Poisson’s ratio and Young’s modulus of the metal material, respectively, \(a\) is the diaphragm radius, and \(d\) is the thickness.

**Fiber optic pressure sensor**

The deflection of the diaphragm center \(\Delta L\) as a function of the applied pressure \(\Delta P\) can be written as:

Reactor Vessel Pressure Instrumentation
Pressure Indication