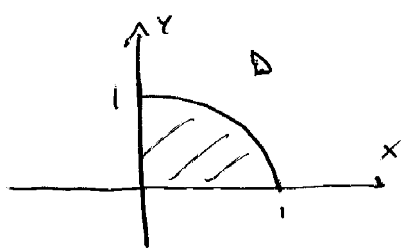
Math 242 Sec. 12 – Fall 2006 Name: SOLUTION

TEST #3

Only one page of notes allowed. No calculators. Please, justify your answers and write clearly if you want credit for your work.

(1) [3 Pts.] Solve the following integral: Sketch the region of integration

$$\int_0^1\int_0^{\sqrt{1-x^2}} 2\,dy\,dx. = 2\int_0^{\pi/2}\int_0^1 r\,dr\,d\theta \quad [1$$



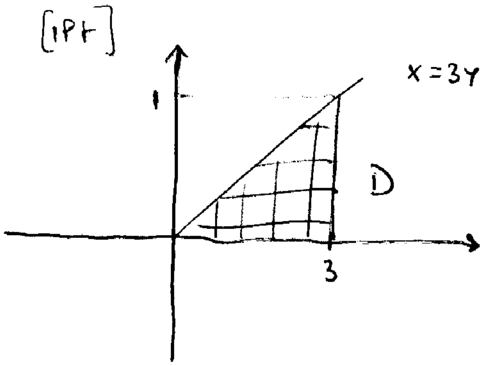
$$= 2\,\frac{\pi}{2}\,\frac{1}{2} = \frac{\pi}{2} \quad [1\,Pt$$

[1 Pt]

(2) [4 Pts] Evaluate the integral by reversing the order of integration. Sketch the region of integration.

$$I = \int_0^1\int_{3y}^3 e^{x^2}\,dx\,dy.$$

[1 Pt]



$x = 3y$ or $y = \frac{x}{3}$

$$D = \{3y \le x \le 3,\ 0 \le y \le 1\}$$
$$= \{0 \le y \le \tfrac{x}{3},\ 0 \le x \le 3\}$$

$$I = \int_0^3\int_0^{\frac{x}{3}} e^{x^2}\,dy\,dx = \int_0^3 \frac{x}{3}e^{x^2}\,dx \quad [2\,Pts$$

set $u = x^2$

$du = 2x\,dx$

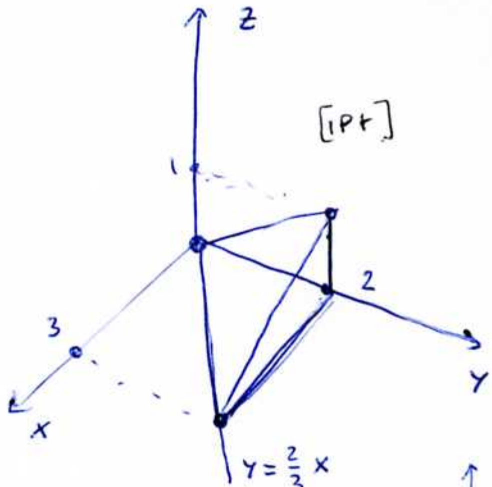
$$= \frac{1}{6}\int_0^9 e^u\,du \quad [1$$

$$= \frac{1}{6}\,e^u\Big|_0^9 = \boxed{\frac{1}{6}\left(e^9 - 1\right)}$$

(3) [4 Pts] Set up (do not evaluate) the integral needed to compute the volume of the solid tetrahedron with vertices $(0,0,0)$, $(0,2,0)$, $(3,2,0)$, and $(0,2,1)$. Sketch the solid.

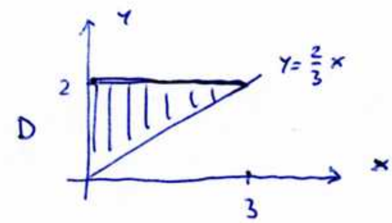
The solid is bounded by the planes

$x=0,\ z=0,\ y=2$ and $z=\frac{y}{2}-\frac{x}{3}$ ← [1Pt]



$$V=\iint_D \int_0^{\frac{y}{2}-\frac{x}{3}} dz\, dA = $$

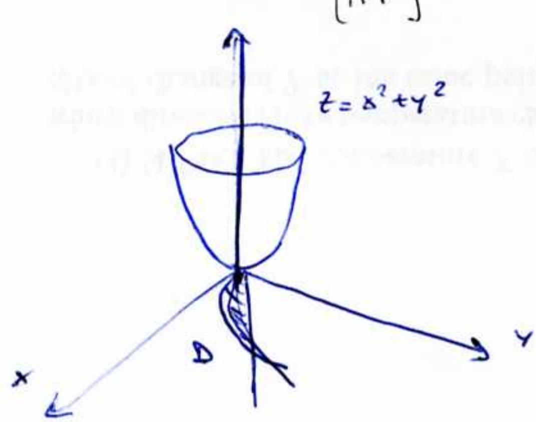
$$=\int_0^2\int_0^{\frac{3}{2}y}\int_0^{\frac{y}{2}-\frac{x}{3}} dz\, dx\, dy$$ [2



$$=\int_0^3\int_{\frac{2}{3}x}^{2}\int_0^{\frac{y}{2}-\frac{x}{3}} dz\, dy\, dx$$

(4) [4 Pts.] Set up the integral needed to compute the volume of the solid under the paraboloid $z=x^2+y^2$ and above the region bounded by $y=x^2$ and $y=x$.

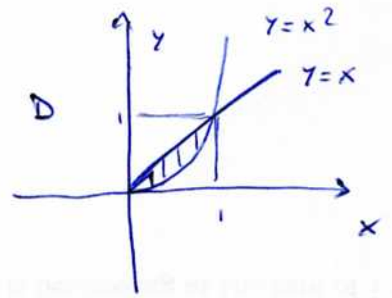
[1Pt]



$$V=\iint_D\int_0^{x^2+y^2} dz\, dA =$$

[3Pts]

$$=\int_0^1\int_{x^2}^{x}\int_0^{x^2+y^2} dz\, dy\, dx$$

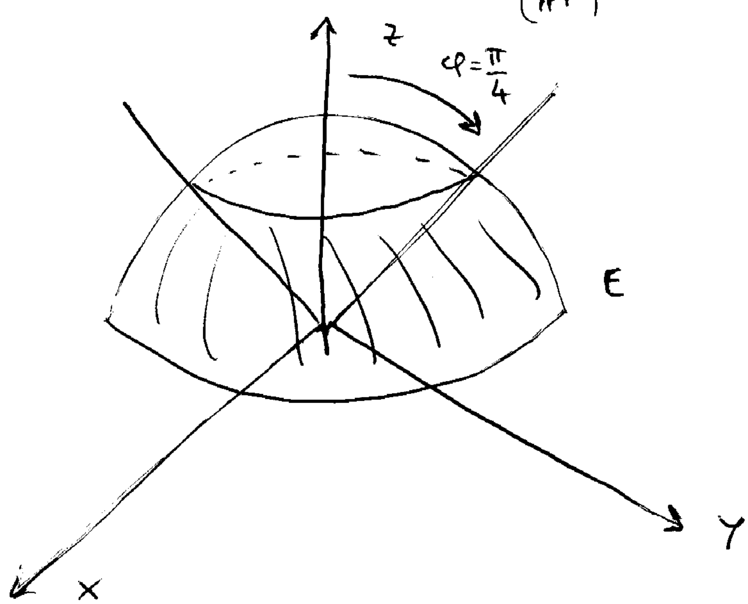


(5) [5 Pts.] Compute the triple integral

$$\iiint_E z\, dV,$$

where E is the solid that lies within the sphere $x^2+y^2+z^2=1$ and below the cone $z=\sqrt{x^2+y^2}$, above the xy-plane. Sketch the solid E. (Hint: use spherical coordinates).

(1pt)



sphere: $\rho = 1$

cone $\varphi = \frac{\pi}{4}$

~~(1pt)~~

$E = \{(\rho, \theta, \varphi) : 0 \le \rho \le 1,\ 0 \le \theta \le 2\pi,\ \frac{\pi}{4} \le \varphi \le \frac{\pi}{2}\}$

$$\iiint_E z\, dV = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^1 \rho\cos\varphi\ \rho^2 \sin\varphi\ d\rho\, d\theta\, d\varphi \qquad \text{(2Ps}$$

$z = \rho\cos\varphi$

$$= 2\pi \int_{\pi/4}^{\pi/2} \sin\varphi\cos\varphi\, d\varphi \int_0^1 \rho^3\, d\rho =$$

$$= 2\pi\ \frac{\sin^2\varphi}{2}\Bigg|_{\pi/4}^{\pi/2}\ \frac{\rho^4}{4}\Bigg|_0^1 = \qquad \text{(2Pts)}$$

$$= \pi\left(1-\frac{1}{2}\right)\frac{1}{4} = \frac{\pi}{8}$$