Math 242 Sec. 7 – Fall 2005 Name: SOLUTION

TEST #2

One single-sided page of notes and a hand calculator allowed. Each problem is worth 4 points. Please, **justify your answers and write clearly if you want credit for your work.**

(1) Find the following limit, if it exists, or show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{8\,xy^2}{x^4+y^4}$$

Set $x=0$. Then $\lim_{(0,y)\to(0,0)} \frac{8xy^2}{x^4+y^4} = \lim_{y\to 0} \frac{0}{y^4} = 0$ (1 Pt)

Set $x=y^2$ Then $\lim_{(y^2,y)\to(0,0)} \frac{8xy^2}{x^4+y^4} = \lim_{y\to 0} \frac{8y^4}{y^8+y^4} = \lim_{y\to 0}\frac{8}{y^4+1} = 8$

Since limit is different for 2 different paths to $(0,0)$, then (3 Pt for correct argument)

LIMIT DOES NOT EXIST

(2) Use implicit differentiation to find dy/dx, where $\cos(x-y) = xe^y$.

let $F(x,y) = \cos(x-y) - xe^y = 0$ (1 Pt)

$$\frac{dF}{dx} = \frac{\partial F}{\partial x}\frac{dx}{dx} + \frac{\partial F}{\partial y}\frac{dy}{dx} = \left(-\sin(x-y) - e^y\right) + \left(\sin(x-y) - xe^y\right)\frac{dy}{dx} = 0 \quad (2\text{ Pt})$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{\sin(x-y) + e^y}{\sin(x-y) - xe^y}} \quad (1\text{ Pt})$$

(3) Find an equation of the tangent plane to the surface $z = \sqrt{x-y}$ at the point $(5,1,2)$.

$f(x,y) = \sqrt{x-y}$

TAN PLANE: $z-2 = f_x(5,1)\,(x-5) + f_y(5,1)\,(y-1)$ (1 Pt)

$f_x(x,y) = \frac{1}{2\sqrt{x-y}}$ $\quad f_x(5,1) = \frac{1}{4}$ (1 Pt)

$f_y(x,y) = -\frac{1}{2\sqrt{x-y}}$ $\quad f_y(5,1) = -\frac{1}{4}$ (1 Pt)

Thus: $\boxed{z-2 = \frac{1}{4}(x-5) - \frac{1}{4}(y-1)}$ (1 Pt)

or $\boxed{x - y - 4z + 4 = 0}$

(4) Find the maximum rate of change of $f(x, y) = \ln(x^2 + y^2)$ at $(1, 2)$ and the direction in which it occurs (express the direction as a unit vector).

$$f_x(x,y) = \frac{2x}{x^2+y^2} \qquad f_x(1,2) = \frac{2}{5} \qquad (1\text{pt})$$

$$f_y(x,y) = \frac{2y}{x^2+y^2} \qquad f_y(1,2) = \frac{4}{5} \qquad (1\text{pt})$$

$$\nabla f(x,y) = \frac{2}{x^2+y^2}(x,y) \qquad \nabla f(1,2) = \frac{2}{5}(1,2)$$

MAX RATE of change $\quad |\nabla f(1,2)| = \frac{2}{5}\sqrt{1+4} = \boxed{\frac{2\sqrt{5}}{5}}$ (1pt)

DIRECTION of max rate of change $\quad u = \frac{\nabla f(1,2)}{|\nabla f(1,2)|} = \boxed{\frac{1}{\sqrt{5}}(1,2)}$ (1pt)

(5) Find the local maximum and minimum values and saddle point(s) of the function $f(x, y) = 10xy - x^2y - xy^2$.

$f_x(x,y) = 10y - 2xy - y^2$ (1pt)

$f_y(x,y) = 10x - x^2 - 2xy$

$f_{xx}(x,y) = -2y \qquad f_{yy}(x,y) = -2x \qquad f_{xy}(x,y) = 10 - 2x - 2y$

CRITICAL POINTS $\quad \begin{cases} y(10-2x-y) = 0 \\ x(10-2y-x) = 0 \end{cases}$

I eq. vanishes at $y=0$, $y = 10-2x$

$y=0$ Into II eq. gives $x=0$ or $x=10$ $\Rightarrow$ $(0,0)$, $(10,0)$ (1pt)

$y = 10-2x$ into II eq. gives $x=0$ and then $y=10$ $\Rightarrow$ $(0,10)$

Also $\begin{cases} 10-2x-y=0 \\ 10-2y-x=0 \end{cases} \rightarrow \left(\frac{10}{3}, \frac{10}{3}\right)$ (1pt)

$D(x,y) = 4xy - (10-2x-2y)^2$

check:

$(0,0) \quad D(0,0) = -100 < 0$

$(10,0) \quad D(10,0) = -100 < 0$ SADDLE POINTS (1pt)

$(0,10) \quad D(0,10) = -100 < 0$

$\left(\frac{10}{3}, \frac{10}{3}\right) \quad D\left(\frac{10}{3}, \frac{10}{3}\right) = \frac{400}{9} - \left(10 - \frac{40}{3}\right)^2 > 0 \qquad f_{xx}\left(\frac{10}{3}, \frac{10}{3}\right) = -\frac{20}{3} < 0$

LOCAL MAX