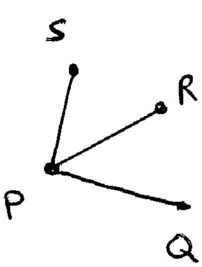
Math 242 Sec. 7 – Fall 2005

Name:

TEST #1

Only one double-sided page of notes allowed. No calculators. Please, justify your answers and write clearly if you want credit for your work. Each problem is worth 4 points.

(1) Find the volume of the parallelepiped with adjacent edges PQ, PR, and PS, where $P(0,2,1)$, $Q(3,2,1)$, $R(-1,3,2)$, $S(2,-1,4)$.



$\vec{PQ} = (3,0,0)$

$\vec{PR} = (-1,1,1)$

$\vec{PS} = (2,-3,3)$

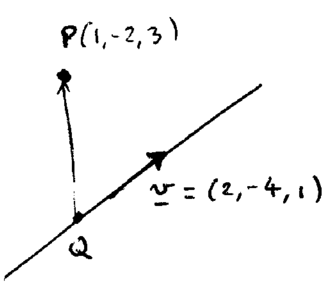
(2 Pt)

$$V = \vec{PQ} \cdot \vec{PR} \times \vec{PS} = \begin{vmatrix} 3 & 0 & 0 \\ -1 & 1 & 1 \\ 2 & -3 & 3 \end{vmatrix} = 18$$

(1Pt) formula

(1Pt) computing determinant

(2) Find the equation of the plane containing the point $P(1,-2,3)$ and the line of equation $\mathbf{r}(t) = (2+2t, -4t, 1+t)$



$Q(2,0,1)$ is a POINT on the LINE

$\underline{u} = \vec{QP} = (-1,-2,2)$ (1Pt)

A normal vector to the plane is:

$\underline{n} = \underline{u} \times \underline{v} = (6,5,8)$ (1Pt)

P is a point on the plane. (1Pt)

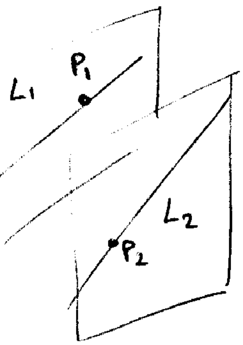
Thus:

$6(x-1) + 5(y+2) + 8(z-3) = 0$ or

$\boxed{6x + 5y + 8z - 20 = 0}$

(1Pt)

(3) Find the distance between the skew lines with parametric equations $x = 1 + t, y = 2 + 6t, z = 2t$, and $x = 1 + 2s, y = 5 + 15s, z = -2 + 6s$.



$$\left.\begin{array}{l} \underline{v}_1 = (1, 6, 2) \quad , \quad \underline{v}_2 = (2, 15, 6) \\ \underline{n} = \underline{v}_1 \times \underline{v}_2 = (6, -2, 3) \end{array}\right) \quad (2\text{Pt})$$

$P_1(1, 2, 0)$ Lies on plane through L_1:

$$6(x-1) - 2(y-2) + 3z = 0$$

$$6x - 2y + 3z - 2 = 0 \qquad \underline{\text{PLANE}} \qquad (1\text{Pt})$$

$P_2(1, 5, -2)$ Lies on line L_2

$$d(L_1, L_2) = d(P_2, \text{plane}) = \frac{|6 - 10 - 6 - 2|}{\sqrt{36 + 4 + 9}} = \boxed{\frac{12}{7}} \qquad (1\text{Pt})$$

(4) Find the curvature of the curve $\mathbf{r}(t) = (t, -t, 1 + t^2)$ ~~at the point $P(1, -1, 1)$~~

$$\left.\begin{array}{l} \underline{r}'(t) = (1, -1, 2t) \\ \underline{r}''(t) = (0, 0, 2) \end{array}\right) \quad (1\text{Pt})$$

$$|\underline{r}'(t)| = \sqrt{1 + 1 + 4t^2} = \sqrt{2}\sqrt{1 + 2t^2} \qquad (1\text{Pt})$$

$$|\underline{r}'(t) \times \underline{r}''(t)| = |2(-1, -1, 0)| = 2\sqrt{2} \qquad (1\text{Pt})$$

$$k(t) = \frac{|\underline{r}'(t) \times \underline{r}''(t)|}{|\underline{r}'(t)|^3} = \frac{2\sqrt{2}}{(\sqrt{2})^3 (1 + 2t^2)^{3/2}} = \boxed{\frac{1}{(1 + 2t^2)^{3/2}}} \qquad (1\text{Pt})$$

(5) Find the length of the curve $\mathbf{r}(t) = (e^t \cos t, e^t \sin t)$ for $0 \leq t \leq 1$.

$$\underline{r}'(t) = (e^t \cos t - e^t \sin t, \; e^t \sin t + e^t \cos t) \qquad (1\text{Pt})$$

$$|\underline{r}'(t)| = \left(e^{2t}\cos^2 t + e^{2t}\sin^2 t - \cancel{e^{2t}\sin t \cos t} + e^{2t}\sin^2 t + e^{2t}\cos^2 t + \cancel{e^{2t}\cos t \sin t}\right)^{1/2}$$

$$= (2e^{2t})^{1/2} = \sqrt{2}\, e^t \qquad (1\text{Pt})$$

$$L = \int_0^1 |\underline{r}'(t)|\, dt = \sqrt{2} \int_0^1 e^t = \boxed{\sqrt{2}\,(e - 1)}$$

(1Pt) (1Pt)