(1) [4 Pts] Let $f, g \in L^1_{loc}(\mathbb{R})$ and suppose that
\[ \int f \phi = \int g \phi \]
for all $\phi \in C^\infty_c(\mathbb{R})$. Prove that $f = g$ a.e.

(2) [4 Pts] (a) Compute the (distributional) derivative of $\chi_{[a,b]}$.
(b) Compute the Fourier transform of the Heaviside function $H(x) = \chi_{(0,\infty)}$. (Hint: write $H(x) = \frac{1}{2} + \frac{1}{2} \text{sgn}(x)$).
(c) Compute the Fourier transform of the locally integrable function $\sin x$. (Hint: recall $\sin x = \frac{e^{2\pi ix} - e^{-2\pi ix}}{2i}$)

(3) [4 Pts] Suppose that $f(x)$ is a bandlimited function with band 1, that is, $\hat{f}(\xi) = 0$ for $|\xi| > 1/2$. Prove that
\[ f(x) = \sum_{n \in \mathbb{Z}} f(n) \frac{\sin(\pi(x-n))}{\pi(x-n)}. \]
This is known as the Shannon sampling theorem.
(Hint: write $\hat{f}(\xi)$ as a Fourier series in $\xi$. Then substitute this Fourier series into $f(x) = \int_{-1/2}^{1/2} \hat{f}(\xi) e^{2\pi i x \xi} d\xi$. )