Math 715 – Spring 2005 Name:

TEST #3

(1)[3 Pts] Is $L^2(\mathbb{R}) \subset L^1(\mathbb{R})$? Is $L^1(\mathbb{R}) \subset L^2(\mathbb{R})$? Prove or disprove by producing a counterexample each one of the two statements.

(2)[3 Pts] Suppose $1 \leq p < \infty$. Show that if $f_n, f \in L^p$ and $f_n \to f$ a.e, then $\|f_n - f\|_p \to 0$ if and only if $\|f_n\|_p \to \|f\|_p$. (Hint: use the generalized Dominated Convergence Theorem in Ex.20, p. 59).

(3)[3 Pts] Suppose f, g are continuous periodic functions.

(a) Show that f is even if and only if $\hat{f}(n) = \hat{f}(-n)$, for each n.

(b) Show that f is odd if and only if $\hat{f}(n) = -\hat{f}(-n)$, for each n.

(4)[3 Pts] Suppose f is a continuous periodic functions with continuous derivative f'. Show that

$$\widehat{f'}(n) = 2\pi i n \, \hat{f}(n)$$

for all $n \in \mathbb{Z}$.