Math 715 – Spring 2005 Name:

TEST #2

(1)[2 Pts] Prove that if $f \in L^+$ and $\int f < \infty$, then, for every $\epsilon > 0$, there exists $E \in \mathcal{M}$ such that $\mu(E) < \infty$ and $\int_E f > (\int f) - \epsilon$.

(2)[3 Pts] Suppose $f_n, f \in L^1$ and $f_n \to f$ a.e. Show that $\int |f_n - f| \to 0$ if and only if $\int |f_n| \to \int |f|$. (Hint: use the generalized Dominated Convergence Theorem in Ex.20, p. 59).

(3)[4 Pts]

(a) Show that if $f_n \geq 0$ and $f_n \to f$ in measure, then $\int f \leq \liminf \int f_n$.

(b) Show that if $|f_n| \leq g \in L^1$ and $f_n \to f$ in measure, then $\int f = \lim \int f_n$ (Hint: use (a)).

(4)[3 Pts] Suppose that $f \in L^1(m)$ (m is the Lebesgue measure). For $a \in \mathbb{R}$ let $f_a(x) = f(x - a)$. Prove that

$$\lim_{a \to 0} \int |f_a - f| \, dm = 0.$$