A Flow-Based Centrality Measure through Resistance Distances in Smart-Grid Networks

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***Abstract*—We propose methods to estimate centrality in Smart-Grid Networks (SGNs) from the view of a flow-based approach. In several network categories, centrality metrics, such as degree, closeness and betweenness, have provided ways to investigate the importance or weakness of components. These well-known metrics utilize either non-global or shortest-path information. We mention several observations which try to use these metrics into a vulnerability measure of SGNs. For this, we stress that using a proper metric, which captures the core network characteristic, is important to induce a correct network analysis. This proper metric changes with network categories. In contrast to data networks, SGNs possess a fundamentally different property that comes from electricity distributions and this requires us to include a multi-path consideration. About this issue, we explain the feasibility of flow-based analysis and suggest to utilize an effective resistance as a distance measure. This allows us to propose new centrality metrics utilizable in SGNs. In several power grid test-beds, our metrics are tested and the differences from using current centrality metrics are compared. These results indicate that SGNs are more scale-free than the estimation from currently used metrics and provide the reason for cascading failure phenomena observed in SGNs. Additionally, we show that the multi-path effect becomes more severe with a network size increment.**

I. Introduction

The process to find out vulnerable points in a network is a fundamental step to make a reliable system. In this reason, the network vulnerability investigation through a connection topology has been done in the literature of Communication, Data and Social networks. A well-known result is the scale-free property of the Internet [1]. The existence of hub nodes whose local degree is extensively high reveals that the structure has a vulnerability to intentional attacks.

Under the name 'centrality', we can find several metrics to investigate the vulnerability of network components. Metrics such as degree, closeness and betweenness are well-known and commonly used [2], [3]. These utilize either local or shortest path information. A prior scale-free property is the representative result by using a degree metric. Closeness and betweenness are cases which utilize shortest paths to estimate the importance of network components (see II-C). In data networks, using shortest paths already have a validity as a tool to measure centrality. For instance, a source utilizes shortest paths to reach destinations by using routing protocols in computer networks and users take information (e.g., news or rumor) propagated through the fastest route in social networks.

Fig. 1. The necessity of multi-path consideration: (a) Incoming data selects one neighbor which is on the shortest path. (b) An incoming power flow is distributed into all neighbors depending on potential differences.

Smart Grid Networks (SGNs), which are structures to make an intelligent electricity distribution in power grids, receive much attentions nowadays. In there, we observe several *cascading network failures* which are critical, but started from a small component failure. For example, the well-known US and Italian black outs in 2003 were caused by a single component failure and broadly spread [4]. Ironically, even under extensive supplements to make a robust network, the reported number of failures does not decrease [5]. We consider this is because of the absence of proper analysis methods.

Already, there exist papers to diagnosis the power network vulnerability by using centrality metrics [4], [6], [7]. However, we stress that choosing a proper metric is a requirement to induce a correct network analysis. This is not invariant to the category of networks. In contrast to data networks, *multi-paths* must be involved in the centrality measure. This is due to the electricity distribution. An incoming current at a node is distributed into all outgoing neighbors and the degree follows the potential difference by Kirchhoff's Law as shown in Fig. 1.

This paper proposes centrality estimation methods, utilizable in SGNs, based on the use of effective resistance. In section 2, we show several network categories which require a multi-path consideration and explain the feasibility of flow-based analysis. Section 3 provides a network conversion to use an effective resistance and proposes new centrality estimation metrics. In several power grid test-beds, we test our metrics and compare the differences from currently used metrics in section 4. After that, we conclude this paper with discussions.

II. Preliminaries

A. Multi-Path Consideration and Flow-Based Approach

First, we refer that the necessity of multi-path considerations is not just limited in SGNs. Suppose that there exists a link-congestion in computer networks or an energy-limitation in wireless sensor networks. In these cases, a source is allowed to select alternative paths, which are not a shortest path anymore [8]. Also, news or rumor in social networks is propagated through non-fastest routes under the denial of forwarders.

Actually, the shortcoming of this shortest path use has been recognized and we can find some alternatives. For instance, a random walk is used to measure the visiting frequency of intermediate nodes between source and destination in [9] and a probabilistic path selection method can be found in [7]. These approaches allow non shortest-paths to appear probabilistically (i.e., shortest-path occurs frequently. Also, other possible paths appear with less frequency).

In a multi-path consideration, we need to distinguish that an electricity is not probabilistically distributed, but deterministically shared. As briefly mentioned, electricity at a node spreads into all neighbors and this requires us to consider all possible paths between any source and destination pair. We give an intuition that a flow-based approach is feasible to capture this: Suppose a complex pipe-line system, where a source and a destination are located at the top and bottom, respectively. When a flow starts at a source, it passes all possible paths to reach a destination and each path contributes to the different amount of quantity delivery. This flow concept can be regarded as the deterministic version of [9], [7].

B. *Effective Resistance and Distance Measure*

We explain that the effective resistance ($\mathbf{R}_e$) [10], [11] can be a candidate metric to support the flow-based estimation. In electrical circuits, this defines an aggregated resistance between source and destination when a potential 1 and 0 are given for them, respectively.

Suppose that we select arbitrary nodes A and B as a source and a destination on a connected graph $G(\mathcal{V}, \mathcal{E})$. When a graph G is given as shown in Fig. 2, the effective resistance $\mathbf{R}_e(A, B)$ is computed by resistance rules in electrical circuits. The value for a graph whose topology has only serial or parallel connections is equal to the sum or the inverse of inverse-sum of each resistance. Especially when the resistance value is same for all edges, the increase of possible paths always induces the smaller effective resistance $\mathbf{R}_e(A, B)$.

This effective resistance is used to measure the correlations between nodes (e.g., when $\mathbf{R}_e(A, B) \downarrow$, the correlation between A and B $\uparrow$). In this reason, we utilize $\mathbf{R}_e$ as a *distance metric* for the centrality measure in SGNs. This raises a scalability issue, as we require $\mathbf{R}_e(s, d)$ values for all (s, d) pairs in $\mathcal{V}$ (see III-B). Under large-scale and complexly connected networks, the computational complexity increases significantly. To relax this problem, we utilize a general relationship between a laplacian graph and an effective resistance.

C. *Metrics for Centrality*

To compare the resulting differences, we introduce three metrics (degree, closeness and betweenness), commonly used in current centrality measure. More details can be found in [2], [3]. When a graph $G(\mathcal{V}, \mathcal{E}, \mathcal{W})$, where $|\mathcal{V}| = n$, is given:

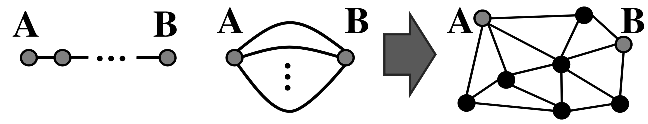


Fig. 2. When n edges exist and each resistance is 1, an effective resistance $\mathbf{R}_e$ is n and $\frac{1}{n}$ for a path and a parallel graph, respectively. *This computation includes all-possible paths between A and B* under potential differences. We use $\mathbf{R}_e$ on general graphs as a distance metric to measure the vulnerability of smart-grid networks.

- **Degree** (C_D): Degree centrality is defined as the number of edges incident upon a node. For a node v, the degree centrality $C_D(v)$ is:

$$C_D(v) = \frac{deg(v)}{n-1} \tag{1}$$

- **Closeness** (C_C): Closeness centrality considers the sum of shortest-distances to all other nodes and the importance of a node becomes higher when the sum of geodesic distances to all other vertices becomes smaller. When we define $d_G(v, t)$ as the shortest-path distance from v to t in G, the centrality of node v ($C_C(v)$), is following:

$$C_C(v) = \frac{1}{\sum_{i \in \mathcal{V} \setminus v} d_G(v, t)} \tag{2}$$

- **Betweenness** (C_B): Betweenness centrality measures the occurrence degree of a node on shortest paths between any node pair. Set σ_{st} and $\sigma_{st}(v)$ are the number of shortest paths from s to t and the number of shortest paths through a node v, respectively. Then, betweenness centrality of node v ($C_B(v)$) is described by:

$$C_B(v) = \sum_{s \neq v \neq t \in \mathcal{V}} \frac{\sigma_{st}(v)}{\sigma_{st}} \tag{3}$$

SGNs are the representative example of a highly-coupled system [12]. In this reason, these metrics have shortcomings to provide centrality in SGNs. A degree metric only uses local information and there is no edge weight distinction. Also, closeness and betweenness only utilize shortest paths and this cannot include the correlation effect that comes from the existence of multi-paths.

III. Flow-Based Vulnerability Measure Approach

We regard that a weighted graph $G(\mathcal{V}, \mathcal{E}, \mathcal{W})$ represents the given network, where $\mathcal{V}$ and $\mathcal{E}$ are a node and an edge set, respectively, and a weight set $\mathcal{W}$ indicates link capacities. Centrality metrics in II-C are computed by using the well-known shortest path algorithms (e.g., Bellman-Ford or Dijkstra) or local degree investigations. In this section, we explain a graph conversion to utilize the effective resistance $\mathbf{R}_e$ on general graphs and propose our centrality metrics utilizable in SGNs.

A. *Graph Conversion into an Electrical Network*

An effective resistance ($\mathbf{R}_e$) is a term in electrical networks. Hence, to use an effective resistance into several network categories, we proceed with a graph conversion from other network categories into an electrical network. This is achieved by modifying edge weights. Suppose that $\mathcal{W}'$ is the modified edge weight set after the electrical network conversion. For data networks and SGNs, we do the following modifications:

- **Data Networks**: Between a *resistance* and a *link capacity* on data network graphs, we describe their inverse proportional characteristic. When there exist multiple neighbors under the flow concept, the higher capacity link delivers a higher amount of flow in the data network. In contrast, the current flow becomes smaller when the resistance value increases. This inverse property lets us set the new edge weight as $w'_i = \frac{\Psi}{w_i}$, where $w'_i \in \mathcal{W}'$ and Ψ is a conversion factor (constant).
- **Smart-Grid Networks**: In the sense of electricity distribution, SGNs partially include the property of electrical networks. When we see the power lines, their impedance $\mathbf{Z}$ is expressed by $\mathbf{Z} = \mathbf{R} + j\mathbf{X}$. The electricity flow is only decided by the reactance value $\mathbf{X}$ under a lossless line assumption and the effect of resistance $\mathbf{R}$ is small enough in reality even under lossy lines [13]. This allows us to set $w'_i = \mathbf{X}_i$, where $w'_i \in \mathcal{W}'$.

From these conversions, the newly constructed graph $G'(\mathcal{V}, \mathcal{E}, \mathcal{W}')$ now represents an electrical network. For this, we consider its laplacian matrix $\mathbf{L}(G')$. In network analysis, this laplacian matrix $\mathbf{L}$ is frequently used to investigate graph properties such as connectivity or convergence speed, as this includes the graph topology information. For any graph G with positive edges, the laplacian $\mathbf{L}(G)$ is defined by:

$$\mathbf{L}_{ij} = \begin{cases} \sum_{j,i \neq j \in \mathcal{V}}^{n} w_{ij} & \text{if } i = j \\ -w_{ij} & \text{if } i \text{ and } j \text{ have an edge} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where w_{ij} is an edge weight between i and j.

In electrical networks, there exists a generalized relationship between an *effective resistance* and a *graph laplacian* [11] and this is defined by the following:

$$\mathbf{R}_e(i,j) = \mathbf{L}^+_{ii} + \mathbf{L}^+_{jj} - \mathbf{L}^+_{ij} - \mathbf{L}^+_{ji} \tag{5}$$

where $\mathbf{L}^+$ is a pseudo-inverse of $\mathbf{L}$. From now on, we use the value $\mathbf{R}_e(i,j)$ as a distance between node i and j as mentioned. By (5), $\mathbf{R}_e$ has a matrix notation with size $|\mathcal{V}| \times |\mathcal{V}|$ and we see the following observation:

Observation 1: [12] For a connected graph (i.e., one cluster), the effective resistance between any node pair is positive (i.e., $\mathbf{R}_e(i,j) > 0$ for $\forall i,j \in \mathcal{V}$ and $i \neq j$). In other words, the graph expression of matrix $\mathbf{R}_e$ becomes a complete graph $K_{|\mathcal{V}|}$.

This observation is reasonably expected when we consider the multi-path effects. As the effective resistance measures a correlation between two nodes, any node pair has at least one path which relates each other under the connected graph assumption. This also can be used as a cluster division indicator as $\mathbf{R}_e(i,j) = \infty$ is guaranteed if no path exists between i and j. The usefulness of relation (5) is that the values of effective resistance among all node pairs can be computed through one-time matrix inversion for general graphs.

	IEEE30	IEEE118	IEEE300	WSCC
Average Degree	2.73	3.03	2.74	2.67
Number of Nodes	30	118	300	4941

TABLE I
TOPOLOGICAL CONNECTIONS OF SEVERAL POWER NETWORKS [14]

B. Flow-Based Centrality Metrics in SGNs

We propose two centrality estimation metrics utilizable in SGNs based on the matrix $\mathbf{R}_e$. These correspond to the concept of degree ($\mathbf{C}_D$) and closeness ($\mathbf{C}_C$) measure, respectively.

1) Degree Corresponding Metric ($\mathbf{S}_1$): We refer to the topological nature of SGNs. Table I indicates that nodes in SGNs have a limited number of local edges ($2 \sim 4$) regardless of the size increment and [14] shows that their connections resemble random graphs when edge weights are ignored.

The degree metric ($\mathbf{C}_D$) only considers the number of local neighbors without edge weight distinctions. Thus, we expect that the analysis of SGNs through $\mathbf{C}_D$ will be similar to a random graph analysis. One property of random graphs is that the importance of a node is close among all nodes [1]. In that, the effect of component failures is almost invariant to its occurrence location. We want to see the differences when multi-paths and edge weight distinctions are considered in the estimation. This edge weight distinction is necessary in SGNs. For instance, the reactance values between inter- and intra-area have huge differences and affect the grid performance [13].

Observation 1 indicates that the matrix $\mathbf{R}_e$, which includes the effect of multi-paths, cannot differentiate the number of local degree due to its completeness (i.e., all nodes have a degree $|\mathcal{V}| - 1$). However, we are aware of their weight differences. From this perception, we deduce a threshold-based filtering on matrix $\mathbf{R}_e$. When a graph $G(\mathcal{V}, \mathcal{E}, \mathcal{W})$ represents the matrix $\mathbf{R}_e$, we execute the following filtering:

$$w'_{ij} = \mathbf{1}_{\{w_{ij} > \tau\}}, \text{ for } (i,j) \in \mathcal{E} \tag{6}$$

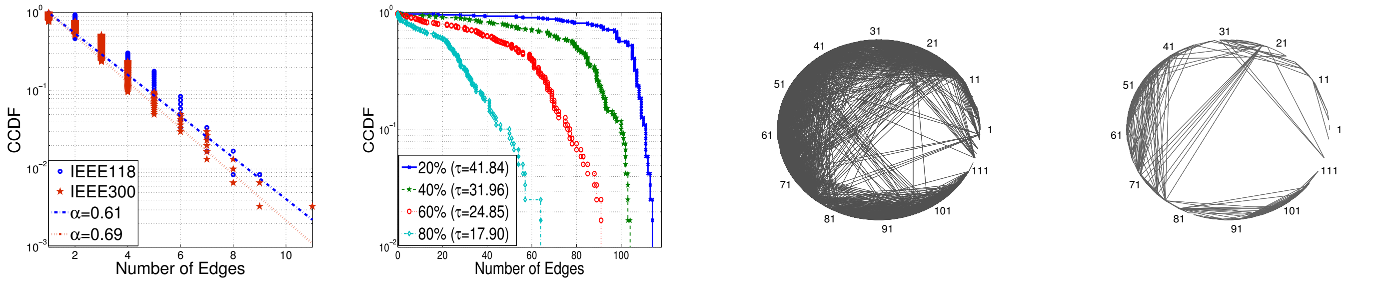
This filtering implies edge removals whose resistance distance is larger than the threshold τ. Thus, if SGNs really possess the random graph property, the disappearance of edges appear symmetrically and the number of remaining edges is also close among nodes after any τ-valued filtering.

We set the local degree of nodes after the filtering to $\mathbf{S}_1$. With varying the value τ, we observe the edge number variation and its removal pattern to see the node importance.

2) Closeness Corresponding Metric ($\mathbf{S}_2$): The previous approach let us see the differences of node importance. However, we still require a metric that provides a *quantified* value for centrality measure. We propose another metric $\mathbf{S}_2$ which corresponds to closeness $\mathbf{C}_C$ by using the elements on $\mathbf{R}_e$:

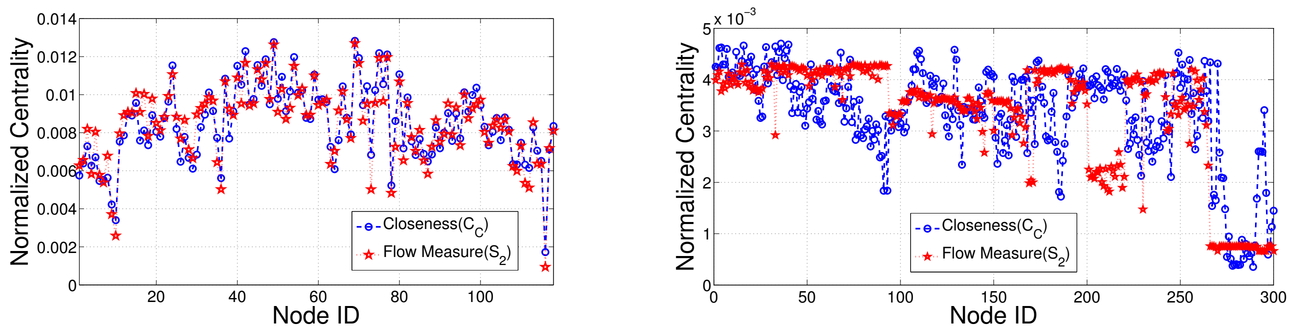
$$\mathbf{S}_2(v) = \frac{1}{\sum_{i \in \mathcal{V} \setminus v} \mathbf{R}_e(v,i)} \tag{7}$$

This has the same form to the closeness measure $\mathbf{C}_C$. The difference is that now we include the effect of multi-paths instead of shortest paths. Each matrix element $\mathbf{R}_e(i,j)$ is the



(a) Degree CCDFs from $\mathbf{C}_D$ (b) Degree CCDFs from $\mathbf{S}_1$ (c) $\tau = 21.3341$ (70% filtered) (d) $\tau = 13.6531$ (90% filtered)

Fig. 3. (a)-(b) are local degree CCDFs (*Semi-Log*) in IEEE 118 Bus. (c)-(d) plot kirk graphs after threshold-based filtering with varying τ. This shows asymmetric edge removal behaviors under the use of metric $\mathbf{S}_1$.



(a) IEEE 118 Bus (b) IEEE 300 Bus

Fig. 4. Normalized centrality comparisons between closeness metric ($\mathbf{C}_C$) and proposed metric $\mathbf{S}_2$ for (a) IEEE 118 and (b) IEEE 300 bus power grids.

distance between i and j which already includes the multiple-path effect. In this reason, result comparisons between $\mathbf{C}_C$ and $\mathbf{S}_2$ give the quantified difference of node importance when multi-paths are considered in centrality estimation. To have a comparison fairness, we normalize measurement values by using $\sum_{i=1}^{n} \mathbf{S}_2(i) = \sum_{i=1}^{n} \mathbf{C}_C(i) = 1$.

IV. Test Cases and Discussions

We test the proposed metrics and currently used centrality metrics in IEEE 118 and 300 Bus power grid test-beds [15]. In power grids, the term 'Bus' represents a node on a graph and each grid has 186 and 411 edges (lines), respectively. Before measuring centrality, we proceed a graph conversion of power grids. For an electrical network to test the proposed methods, we assign edge weights to $w_{ij} = \mathbf{X}_{ij}$ (i.e, line reactance) and set the graph to G_1. To use the centrality metrics in section II-C, we set edge weights to $w_{ij} = \frac{\Psi}{\mathbf{X}_{ij}}$ and define its graph as G_2. The conversion constant (Ψ) is set to 1 for convenience.

A. Degree-Based Tests

Under the use of degree metric ($\mathbf{C}_D$), we verify that its use induces a random graph property for SGNs. The investigation of local edge degree CCDF is one way to see it. Simply, the topology mimics a random graph when its degree CCDF follows an exponential function (i.e., $e^{-\alpha x}$). For IEEE 118 and 300 Bus, Fig. 3(a) plots the local edge degree CCDFs in *semi-log* scale. We observe that they are well-approximated to dotted lines (i.e., exponential) with $\alpha = 0.61, 0.69$.

We test the proposed degree corresponding metric ($\mathbf{S}_1$), including edge weight differences and multi-paths. The effective resistance matrix $\mathbf{R}_e$ is computed by using G_1 laplacian and eq. (5). Then, we use the threshold filtering (6) on $\mathbf{R}_e$ with varying τ.

We observe the edge removal pattern through a Kirk graph expression, where the location of nodes is uniform on a circle. This allows us to notice the edge removal behavior with ease. For IEEE 118 Bus system, Fig. 3(c) and (d) show kirk graphs when 70% and 90% edges are filtered by using different τ. In there, the remaining edges mean the stronger correlation between buses than the disappeared.

These figures show that the occurrence of edge removals is highly *asymmetric*. We plot only two high edge reduction cases due to the eye-verification difficulty of dense graph under low edge reduction (i.e., large τ). However, the same asymmetric removal pattern appears over all τ ranges for both IEEE 118 and 300 Bus. For more supports, Fig 3(b) plots the CCDF of local edges under $\mathbf{S}_1$ with varying τ. These *semi-log* CCDFs indicate a power-law distribution regardless of τ selections. In topological sense, this corresponds to a *scale-free* graph. Under the consideration of multi-paths and edge-weights, our metric shows *opposite* graph properties to the degree metric $\mathbf{C}_D$ in centrality.

B. Closeness-Based Tests

We compare the normalized centrality between the proposed $\mathbf{S}_2$ and closeness metric $\mathbf{C}_C$. In the measurement procedure, these two have a totally different edge weight set. Fig. 4(a) and (b) plot the normalized centrality under IEEE 118 and 300 Bus systems, respectively. We make the following observations:

1) Envelop Similarity: In Fig. 4(a), we still observe an envelop similarity for IEEE 118 bus although the centrality estimation between two metrics use totally different edge weight sets. This can provide an indicator whether the closeness metric, used in data networks, is still valid in power networks. In other words, this comparison supports the importance of shortest-paths in SGNs. Unfortunately, this envelop similarity disappears under the increment of network size. Fig. 4(b) plots the envelop disagreement in IEEE 300 Bus system.

2) Node-Level Centrality Difference: Previous envelop comparison provides only rough observations. In that, we plot the centrality ratio (i.e., $\mathbf{S}_2/\mathbf{C}_C$) to see more details in Fig. 5. Simply, when the ratio is 1 for a node, this means both metrics evaluate the node with same importance (or vulnerability). This node level comparison reveals that there exists a node whose centrality has a 83% difference in IEEE 118 Bus and the difference becomes severe with the increasing size of power grids. In IEEE 300 Bus, we find that the estimated centrality can be different more than 600%.

C. Lessons and Discussions

1) ***Scale-Free Property****:* Section IV-A and [14] show that the analysis of SGNs through their physical connection topology follows the property of random graphs. However, this brings a contradiction regarding the cascading failure phenomena in SGNs. These are more likely to occur in scale-free graphs. We suspect that there is a missing link to capture the correct characteristics of SGNs and add the intrinsic electrical property in the centrality measure by considering multi-paths. This allows us to see the existence of scale-free properties.

2) ***Multiple-Path Effects****:* The recent trend of power grids has been changing from a *radial* to an *inter-connected* system [13]. Additionally, this accompanies the increment of grid size. For instance, the WSCC network has more than $4,000$ buses (see Table I). This trend seriously increases the number of paths among buses. From the comparison between IEEE 118 and 300 bus system, we show the necessity and importance of multiple-path considerations in SGNs. This must not be overlooked, as future power grids are expected to be more larger scale and complexly connected.

3) ***Extension to Other Network Categories****:* We see several network categories that require the multiple-path consideration. Our flow-based approach based on the use of an effective resistance provides an alternative method for those cases (e.g., a flooding, link-congestion, fair-energy consumption routing in data networks [8], and several word-of-mouth propagations in social networks [9]). In contrast to probabilistic approaches which use a *random walk* or a *random path* [9], [7], the proposed metrics provide methods to estimate multi-path effects deterministically for general topologies.

V. Conclusion

In this paper, we proposed flow-based centrality estimation methods for power grids. This is initiated from a proper metric necessity to induce a correct SGN analysis. The fundamental network property difference which comes from electricity distribution made us consider the effect of multiple-paths in SGNs. About this issue, we explained the feasibility of flow-based approach and utilized an effective resistance to propose centrality metrics utilizable in SGNs. When this is rewarded, we observed the more scale-free property which can explain cascading failure phenomena. Additionally, we showed that the effect of multi-paths became severe with the size increment of power grids. With acknowledging that the recent trend of power grids is to become larger in scale and highly interconnected, the proposed flow-based analysis is essentially required in the design and analysis of SGNs.

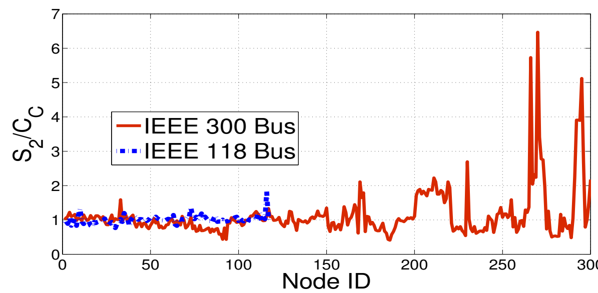


Fig. 5. A ratio $\mathbf{S}_2/\mathbf{C}_C$ for IEEE 118 Bus and IEEE 300 Bus networks.

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