# Towards Improved Scalability in Smart Grid Modeling: Simplifying Generator Dynamics Analysis via Spectral Graph Sparsification

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# The Analysis Complexity of Generator Dynamics

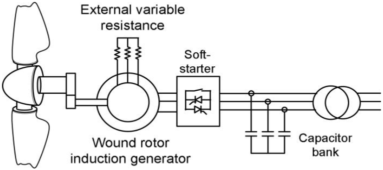
- **Objective:**
  Reduce the analysis complexity of generator dynamics in power grids.
- **Reasons for the complexity**

1. **Non-Linear Behavior of generators**:

- 2nd-order *swing equation* model (The simplest one)

$$\dot{\delta}_i = \Omega\omega_i,\ 2H_i\dot{\omega}_i + D_i\omega_i = P_{mi} - P_{ei},\ i = \{1, \ldots, n\} \quad (1)$$

$\omega_i$: Rotor Speed, $\delta_i$: Rotor Angle, $D_i$: Damping Coefficient
$P_{mi}$: Mechanical Power Input, $P_{ei}$: Electrical Power Output
$\Omega$: Conversion Factor $((p.u) \rightarrow (rad/s))$, $H$: Machine Inertia.

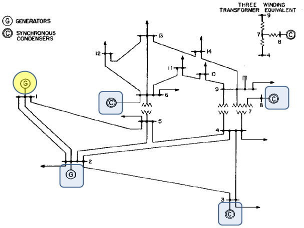



(e.g., Wind generator)

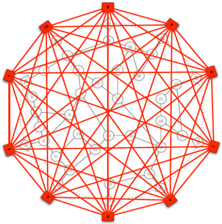
# The Analysis Complexity of Generator Dynamics (Cont')

2. **Correlations among generators**:
   The dynamics of one generator is affected by all other generators in the grid.
   - In (a), the dynamics of yellow generator has correlations with all other blue generators.
   - Measure the correlation among generators: In (b), Kron reduction provides a mathematical way to reduce algebraic buses.

→ In this procedure, the topology connection among generators always becomes a *complete* graph.




(a) IEEE 14 Bus System



(b) Kron Reduction Example

# The Analysis Complexity of Generator Dynamics (Cont')

- Under a power grid with $n$ generators, the reasons 1 and 2 require us to solve:

**FULLY CORRELATED $n$ NON-LINEAR DIFFERENTIAL EQUATIONS!**

- Mathematically, this is given by:

$$\ddot{\delta}_i = \frac{1}{2H_i}(P_{mi} - \sum_{j=1}^{n} c_{ij} \sin(\delta_i - \delta_j)), \quad i \in \{1, \ldots, n\} \tag{2}$$

- **Small-Signal Stability Analysis** (Traditional approach):
  - Consider the Jacobian (i.e., Linearized system matrix) of (2) at operating points.
  - Then, investigate its eigenvalues.

$\rightarrow$ Real part: GENERATOR STABILITY, Imaginary part: SYSTEM MODES

# Power Grids: Recent Trends

- **TREND 1**: The size increment of power grids.

| | IEEE30 | IEEE118 | IEEE300 | NYISO | WSCC |
|---|---|---|---|---|---|
| Average degree | 2.73 | 3.03 | 2.73 | 4.47 | 2.67 |
| Number of buses | 30 | 118 | 300 | 2935 | 4941 |

2. **TREND 2**: The more distributed electricity generation.

- Utilizing renewable energy generation (e.g., solar, wind generators)

$\rightarrow$ The number of generators in grids $\uparrow$

$\rightarrow$ The correlation importance among generators $\uparrow$.

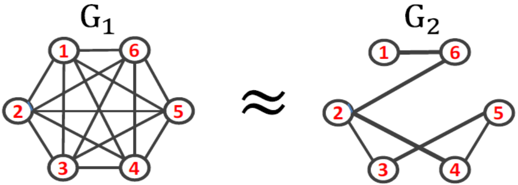
Small-Signal Stability Analysis becomes too complex under these trends

Can we reduce the complexity for the analysis of generator dynamics?

# Graph Density Problem for Eigenvalues

- **In small-signal stability analysis:**
  - We need the **eigenvalues** of system matrix to investigate the dynamics of generators
  - Kron reduction (i.e., remove *algebraic buses*) induces the matrix full all the time (i.e., **complete graph**).

$\rightarrow$ As the grid size and the number of generators $\uparrow$, complexity for eigenvalues highly increases.




- For the complete graph, there can exist a *sparse counterpart* which has close spectral properties.
- [Advantage]: Can reduce computations for eigenvalues a lot!

# Spectral Property Comparison between Graphs

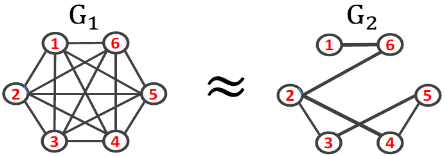
- Set Laplacian matrices of two graphs $G_1$ and $G_2$ as $\mathbf{L}$ and $\bar{\mathbf{L}}$, respectively, and consider the following condition:

$$\forall x \in \mathcal{R}^n \quad (1-\epsilon)x^T\mathbf{L}x \le x^T\bar{\mathbf{L}}x \le (1+\epsilon)x^T\mathbf{L}x \tag{3}$$

- From Courant-Fisher Theorem, an eigenvalue is defined by:

$$\lambda_i = \max_{S:dim(S)=k} \min_{x\in S} \frac{x^T\mathbf{L}x}{x^Tx} \tag{4}$$

- The combination of (3) and (4) means:
- Each corresponding eigenvalue is bounded by an error-bound $\epsilon$
- Is there a simple (i.e., fast) method to satisfy the condition (3)?

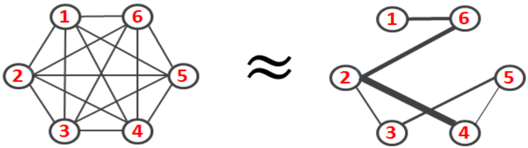


# Spectral Sparsification

- **If we cut some edges on the graph, it implies that:**
- The edge corresponding generator correlations are ignored.

→ Under this, the obtained generator dynamics may not capture the correct one.

- *[IDEA]*: **Weight Control of Remaining Edges**
- After removing edges, the spectral property of a sparsified graph can be preserved by the edge control of remaining edges




- Spielman et al [STOC'08]:
- A random-sampling algorithm which satisfies the condition (3).
- It only has a nearly-linear complexity for sparsification.

# Extension to Power Grid Analysis

- In power grids, using the sparsifier does have the following **challenges**:

1 **Time-varying property of the linearized system matrix**

- For generator dynamics, do we need the sparsification at every analysis instance?

2 **Sub-Matrix Sparsification Problem in Stability Analysis**

- In general, the sparsification of sub-matrix does not conserve the spectral property of whole matrix (some exceptions occur when all sub-matrices are Hermitian).

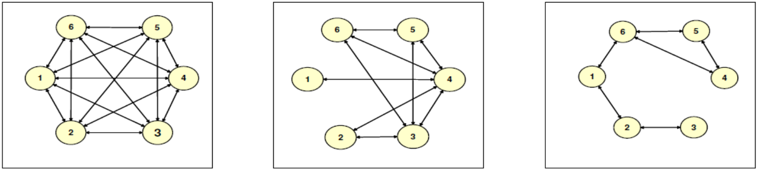
→ Instead of utilizing the sparsifier on the linearized system matrix, we use it to sparsify the grid topology (i.e., reduced admittance matrix).

→ This approach resolves the challenges (Please, see our paper for details).

# Results for Connection Topology Sparsification Test

**Some sparsification results for IEEE 30 Bus System:**

→ 6 generators. With varying the sampling degree, we plot its topology variation by using Graviz tool in MATLAB.




(a) Reduced Admittance $\mathbf{Y}_R$ (b) 40% sparsified ($q = 10$) (c) 60% sparsified ($q = 8$)

→ After the sparsification, the weights of remaining edges are autonomously adjusted (to preserve spectral properties)

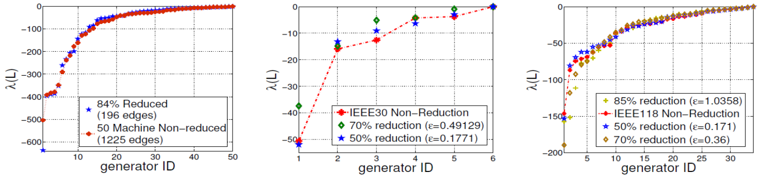
→ Weight matrices for Fig. (a) and (c):

$$adj(\mathbf{Y}_R) = \begin{pmatrix} 0 & 20.64 & 0.34 & 1.87 & 0.21 & 0.45 \\ 20.64 & 0 & 6.24 & 5.45 & 0.56 & 0.81 \\ 0.34 & 6.24 & 0 & 2.84 & 0.27 & 0.25 \\ 1.87 & 5.45 & 2.84 & 0 & 1.53 & 1.42 \\ 0.21 & 0.56 & 0.27 & 1.53 & 0 & 0.66 \\ 0.45 & 0.81 & 0.25 & 1.42 & 0.66 & 0 \end{pmatrix} \quad adj(\mathbf{Y}_R^{60\%}) = \begin{pmatrix} 0 & 43.09 & 0 & 0 & 0 & 1.84 \\ 43.09 & 0 & 5.63 & 0 & 0 & 0 \\ 0 & 5.63 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.84 & 1.99 \\ 0 & 0 & 0 & 1.84 & 0 & 1.32 \\ 1.84 & 0 & 0 & 1.99 & 1.32 & 0 \end{pmatrix}$$

# Eigenvalue Test for System Modes and Stability

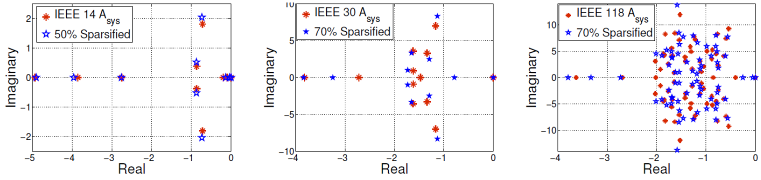
## System Mode Comparisons after Sparsifications:

- For IEEE 30, 118 Bus Systems and 50 Machine System




(a) 50 Machine System (b) IEEE 30 BUS (c) IEEE 118 BUS

## Stability Comparisons after Sparsifications:

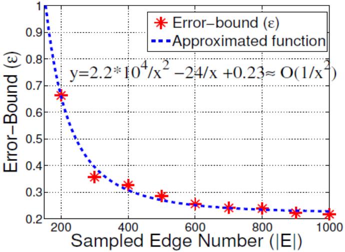



(a) IEEE 14 BUS (b) IEEE 30 BUS (c) IEEE 118 BUS

(For each test, its sparsification error-bound $\epsilon$ is shown on our paper)

# The Degree of Complexity Reduction and Error-Bound ($\epsilon$)

- With varying the sampling degree ($q$) (i.e., the computational complexity), we measure the eigenvalue error-bound ($\epsilon$) for 50 Machine System.
- From Spielman [STOC08], it is defined by $|E'| = O(n \log n/\epsilon^2)$ and $|E'| \leq q$.




→ The tighter error-bound with increasing the number of samplings.

→ Allow us to set-up a sparsification level (i.e. Trade-offs between a complexity and an accuracy).

# Lessons

- **We reviewed the mathematical complexity to investigate generator dynamics in power grids.**
  - For generator dynamics, there exists a mathematically tractable method (Small-signal stability analysis).
  - However, the network topology and the non-linearity of generator behaviors raise challenges.
- **The proposed sparsification approach gives a huge complexity reduction while investigating generator dynamics.**
  - → The error-bound ($\epsilon$) is still small even under high sparsification rates.
  - → The error becomes more tighter with the increase of power network size.
  - → We even can control the error-bound (trade-off between a complexity and an error).
  - → Applicable not only for system mode detection, but also for the stability analysis.