Rigid-Body Image Registration using Mutual Information

Cranos Williams
BAE 790I - Image Processing Project 1
Professor: Dr. D. Lalush
Date Due: 4/30/04
Email: cmwilli5@ncsu.edu

Abstract

Mutual information is a technique that has been successfully used as a comparison function for registering images. It is a measure of how much information one image contains about another. In this paper, mutual information has been used as an objective function for finding the best transformation that will align the misregistered image with the original image. This work is restricted to rigid affine transforms only (e.g. rotation, translation, scaling, and skew). The parameters yielding the maximum mutual information will be the parameters that best register the misregistered image. This algorithm is implemented in Matlab and tested using different protocols of MRI images.

I. INTRODUCTION

Medical imaging diagnostics has been a growing area of research in the past twenty years. Algorithms designed to assist medical physicians in diagnosing patients have grown at an alarming rate. In many cases, it is important to obtain useful data about key regions of separate medical images. The first step in obtaining this information involves the spatial alignment of these images. This process is known as image registration. Image registration is the act of spatially mapping the coordinate system of one image to the coordinate system of another image. Image registration techniques can be partitioned into two main categories: rigid and non-rigid image registration techniques. Rigid registration takes into consideration any global transformation differences between two or more images. This normally involves fewer than ten degrees of freedom. Non-rigid registration takes into consideration any local deformations that could occur due to tissue movement, for example. Non-rigid registration normally requires many more degrees of freedom than rigid registration. The process of registering two or more images can be computationally intensive. The general procedure, however, is relatively straight-forward. The algorithm can be stated as follows:

1) Given two images to be registered, one should denote one as the reference image and the other should denote the mis-registered floating image. The objective is to transform the floating image until it looks like the reference image.
2) Choose a criterion function that will determine the degree of match/mismatch between the reference image and the transformed floating image. Also choose a stopping criterion that indicates the images are registered.
3) Optimize the transformation on the test image such that the stopping criterion is met.

The choice of criterion function is an essential part of the registration process. Comparison methods such as cross-correlation and mutual information are some of the more common techniques found in the literature. Correlation techniques perform well in mono-modal registration wherein there is a linear relationship between the measurements for the same spatial elements in the two image acquisitions [1]. However, because of the non-linear relationship that can arise between the intensities of images across different modalities, correlation has been shown generally not to be a suitable candidate for a criterion function in multi-modal image registration. Since its introduction in 1995 by Viola and Wells [3], mutual information has been one of the most discussed and (usually) acclaimed registration measures for multi-modal image registration [2]. Mutual information is a statistical measure that assesses the strength of dependence between two stochastic variables. Even though mutual information has been shown to outperform other comparison methods used for registration, it is not a panacea [4]. The process can often be improved by incorporating spatial information when performing the alignment [4]. This, however, is beyond the scope of this report.
In this paper, I plan to discuss how mutual information can be used to perform rigid-body image registration. Section II gives theoretical background on mutual information. Section III gives the general process for performing rigid-body image registration. This section also discusses how registration is accomplished using mutual information as the criterion function. Section IV shows mono-modal and multi-modal registration results that were obtained by using mutual information. Section V gives the conclusion.

II. Background Theory

Mutual information (MI) is a statistical measure that finds its roots in information theory. MI is a measure of how much information one random variable contains about another. The MI of two random variables A and B can be defined as

\[
I(A, B) = \sum_{a,b} p_{A,B}(a,b) \log \frac{p_{A,B}(a,b)}{p_A(a) \cdot p_B(b)}
\]

where \( p_{A,B}(a,b) \) is the joint probability mass function (pmf) of the random variables A and B, and \( p_A(a) \) and \( p_B(b) \) are the marginal probability mass functions of A and B, respectively. In working with images, the functional form of the pmf is not readily accessible. The normalized histograms of the intensity values for each image serves as a good approximation of the pmf. The MI can also be written in terms of the marginal and joint entropy of the random variables A and B as follows

\[
I(A, B) = H(A) + H(B) - H(A, B)
\]

where \( H(A) \) and \( H(B) \) are the entropies of A and B, respectively, and \( H(A, B) \) is the joint entropy between the two random variables. They are defined as

\[
H(A) = -\sum_a p_A(a) \log p_A(a)
\]

\[
H(A, B) = -\sum_{a,b} p_{A,B}(a,b) \log p_{A,B}(a,b)
\]

One interpretation of entropy is as a measure of uncertainty of a random variable. A distribution with only a few large probabilities has a low entropy value; the maximum entropy value over a finite interval is achieved by a uniform distribution over that interval [4]. The entropy of an image indicates how difficult it is to predict the gray value of an arbitrary point in the image. MI is bounded by cases of either complete dependence or complete independence of A and B, yielding values of \( I = H \) and \( I = 0 \), respectively, where \( H \) is the entropy of A or B.

The strength of the mutual information similarity measure lies in the fact that no assumptions are made regarding the nature of the relationship between the image intensities in both modalities, except that such a relationship exists. This is not the case for correlation methods, which depend on a linear relationship between image intensities. For image registration, the assumption is that maximization of the MI is equivalent to correctly registering the images. It is clear in Eq. 2 that if the joint entropy of A and B are not affected by the transformation parameters, maximizing the MI is equivalent to minimizing the joint entropy. The joint entropy is minimized when the joint pmf of A and B contain few sharp peaks. This occurs when the images are correctly aligned. When the images are misregistered, however, new combinations of intensity values from A and B will cause dispersion in the distribution. This dispersion leads to a higher joint entropy value, which in turn decreases the MI [2].

III. Rigid-Body Image Registration

A. Performing Image Registration

The process of image registration refers to the procedure of geometrically aligning the coordinate system of two or more images. This paper concentrates only on the registration of two images. Given two images, one image is selected as the reference image \( R \), and the other is selected to be the floating \( F \). The floating image \( F \) is transformed by some linear transformation until it is spatially aligned with the reference image \( R \). Let \( T_{RF} \)
be a linear transformation with the vector parameter $\bar{a}$. The number of elements in $\bar{a}$ determines the degrees of freedom. For this 2-D application, an affine transformation with six degrees of freedom was adequate to perform the registration. The transformation is given as

$$F' = T_{\bar{a}}(F)$$

(5)
or

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

(6)

where $(x, y)$ are coordinates in the floating image $F$ and $(x', y')$ are coordinates in the transformed floating image $F'$. It is obvious that the selection of $T_{\bar{a}}$ is not restricted to the transformation matrix given in Eq. (6). Other transformation matrices can be used based on the assumptions made regarding the nature of the mis-registration.

An appropriate criterion function which is able to determine the degree of mismatch between the reference image $R$ and the transformed floating image $F'$ must be chosen. MI was chosen as an adequate function based on its previous success in the literature as being a good measure for multi-modal applications. MI is a popular measure of comparison because of its ability to measure the similarity between the two images while somewhat ignoring the mismatch of intensities across modalities. For a given reference image $R$, floating image $F'$, and transformation $T_{\bar{a}}$, the MI is calculated by

$$I(R, F', T_{\bar{a}}) = \sum_{r, f'} p_{R,F}(r, f') \log \frac{p_{R,F'}(r, f')}{p_R(r) \cdot p_{F'}(f')}$$

(7)

where the transformation that correctly registers the images is given by

$$T_{\bar{a}_{\text{reg}}} = \arg \max_{T_{\bar{a}}} I(R, F', T_{\bar{a}})$$

(8)
or

$$T_{\bar{a}_{\text{reg}}} = \arg \min_{T_{\bar{a}}} -I(R, F', T_{\bar{a}})$$

(9)

After $T_{\bar{a}_{\text{reg}}}$ is found, it is then applied to the floating image $F$ to produce $F'_{\text{reg}}$. The reference image $R$ and the registered floating image $F'_{\text{ref}}$ are compared to test how well the registration process performed.

B. Calculating MI using Pixel Intensity Values

The first step in the registration process was to make an initial guess for the registration transformation. The help in avoiding local minimums and sped up the convergence of the optimization. The initial guess for the parameters were made based on a rough alignment of the spatial eigenvectors using principle component analysis. With this technique, the rotation was estimated by finding the difference between the principle component angles. The initial value of the translation in the $x$ and $y$ directions was estimated by comparing the center of gravity (COG) of the two images. The initial scale in both the $x$ and $y$ directions were set to 1. Given the initial guess

$$\bar{a}_0 = \begin{bmatrix} a_{11} & a_{12} & a_{21} & a_{22} & t_x & t_y \end{bmatrix},$$

(10)
an optimization routine was used to find the maximum value of $I$ (or minimum value of $-I$) for the transformation $T_{\bar{a}}$ as shown in Eq. (8) and Eq. (9). The MI given in Eq. (7) is calculated by generating a normalized joint histogram based on the intensities of both the reference image $R$ and the transformed floating image $F'$. Local minima in the criterion function were eliminated by blurring the joint pmf as well as blurring the images before calculating $I$. This prevented irregular jumps in the MI function. It was also important to find a function that would make the MI smooth at the minimum. The following function was used to ensure that the global minimum of the optimization function had no irregularities.
\[ I' = \exp(-I^2) \]  

(11)

It is clear that this transformation does not change the structure of the original MI function. Figures 1 and 2 show plots of \(-I\) and \(I'\) with respect to changing rotation.

![Fig. 1. -I w.r.t Changing Rotation](image1)

![Fig. 2. I' w.r.t. Changing Rotation](image2)

IV. RESULTS

The two images that were used to test the registration algorithm described above are shown in Figures 3 and 4. These are both MRI images from the same subject but different protocols.

![Fig. 3. Spin-Density Axial MRI Image](image3)

![Fig. 4. T1 Axial MRI Image](image4)

A. Spin-Density vs. Spin-Density

The registration algorithm was first performed on the MRI image in Figure 3. The original image was labeled as the reference image \(R\). The mis-registered form of that image was labeled as the floating image \(F\). The mis-registered image is shown in Figure 5. This image is misregistered by a rotation of 15°, translation of 3 and 3 pixels in the \(x\) and \(y\) directions, and scaling of .7 and .8 in the \(x\) and \(y\) directions.

Using the criterion function in Eqs. (7) and (9), along with the optimization routine described in above, \(T_{\text{reg}}\) was calculated. This transformation was then applied to the mis-registered image in Figure 5, yielding the image \(F'_{\text{reg}}\).
This image is shown in Figure 6 along with the sum of squared differences of the original image and the registered image in Figure 7. It is evident that there are only small differences in intensity values between the original image and the registered image. In fact, these differences in the intensities can be attributed to the sub-pixel interpolation of the intensity values. Otherwise, the two images are geometrically and spatially aligned, yielding a correctly registered image.

**B. Spin-Density vs. T1**

The registration algorithm described above was also performed on using both MRI images. In this case, the original image in Figure 3 was labeled as the reference image $R$ and the mis-registered form of Figure 4 was labeled as the floating image $F$. The optimal transformation, $T_{GR}$, was found using the optimization in Eq. (9). The mis-registered image is shown in Figure 8. In this case, mis-registration was given by a rotation of $-20^\circ$, translation of 3 and 5 pixels in the $x$ and $y$ directions, and scaling of .8 and .9 in the $x$ and $y$ directions. The registered image is shown in Figure 9. In this case, the absolute difference was not calculated. It is apparent, however, that the registered T1 image is fully aligned with the reference image in Figure 3. These results are particularly interesting given the fact that the image was registered using an image generated by a different protocol.

**V. Conclusion**

In this paper, a brief overview of mutual information and how it can be used to perform image registration was given. It was also shown that based on its properties, it is a useful method for registering images that do not have the same intensity mapping. The traditional way of calculating MI involves generating a normalized joint histogram using only intensity values in the image. In some cases, this may put limitations on the algorithms ability to
correctly align the spatial characteristics of two images. Without this information, the algorithm still performs very well. According to most of the current literature, MI is one of the more robust methods for registering multimodal images.

REFERENCES