Adaptive Online Battery Parameters/SOC/Capacity Co-estimation
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Abstract— Total capacity is one of the most important parameters to characterize the performance and application of a battery. Although the nominal capacity is provided by the manufacturer, the actual capacity is subject to change with cycling effect, temperature and even storage ageing of the battery. Following our previous publications in which we developed an online adaptive parameters/state of charge (SOC) co-estimation algorithm to identify the parameters of the dynamic model of the battery and accordingly design an observer to estimate the SOC. In this paper, first we show that the parameters identification and SOC estimation results are not dependent on the correct approximation of the capacity. Afterwards, using the estimated SOC, we design another observer to estimate the actual capacity of the battery.

Keywords— Battery, Parameters Identification, SOC estimation, Capacity estimation, SOH

I. INTRODUCTION

Advanced battery technology serves electric vehicles industry with employing different chemistries and assembling techniques to provide higher power and energy density. Nonetheless, the mere utilization of these technologies does not guarantee the efficiency, safety and reliability of the battery function. To ensure these features, the battery’s status needs to be accurately monitored and controlled by the algorithms that are designed to perform battery management system (BMS) [1]. The total capacity is one of the most crucial characteristics of the battery that needs to be monitored. All of the methods that rely on the coulomb counting to estimate the State of Charge (SOC) need to have an accurate estimation of the total capacity. Moreover, the full capacity and its degradation due to ageing is a prominent indicator to determine the State of Health (SOH) of the battery. Other than ageing in the form of cycling or storage ageing, the ambient temperature can also cause capacity fading that makes the total capacity of the battery different from the nominal capacity.

Different methods have been used so far to measure the capacity of the battery. Among those, the most direct one is to count the electrons (coulomb counting) when the fully charged battery is discharged with a small current [2]. This method usually used in cycling tests is time consuming, damaging to the battery and not suitable for online applications. Other practical approaches to estimate the capacity are based on the estimated SOC of the battery. Typically, in these methods the SOC is estimated by an algorithm that is not relied on the coulomb counting; and the capacity of the battery is determined by comparing those results to the coulomb counting formula in the interval between estimations. Considering this general guideline, some of the studies use Extended Kalman Filter (EKF) [3], [4] to estimate the SOC and the capacity simultaneously while some of them use artificial intelligent algorithms [5], [6] to determine the capacity based on the SOC. The main issue of these methods is that since the accuracy of the SOC estimation significantly affects the capacity estimation, the model with non-updating parameters does not provide high resolution SOC estimation. Moreover, while most of the artificial intelligent algorithms are not suitable for online applications, considering the capacity as a variable state of the battery model in the simultaneous methods makes the model complicated with non-negligible nonlinearity.

In this paper, we use the results from our previous publications [7], [8], [9] on the battery parameters/SOC co-estimation to calculate the capacity. In [7]-[9], we proposed an online adaptive method to identify the parameters of the battery and estimate the SOC by designing an observer with updating parameters. In this paper, we first show that considering the nominal capacity instead of the actual capacity in the battery model does not affect the parameters identification and the SOC estimation results. Therefore, we can use the SOC estimation results as a benchmark to find the actual capacity. As a supplementary result, the identified parameters are valid to be used to perform the battery modeling and for SOH estimation analysis, since we know that the internal resistance is another important factor to estimate the SOH. We then design another observer with the coulomb counting equation to estimate the actual capacity of the battery. After verifying the designed observer with the simulated results, we apply the capacity estimation method to experimental data from a lithium polymer battery. Since the continuously updating structure of the parameters/SOC co-estimation enhances the accuracy of the estimated SOH, the corresponding capacity estimation gives significantly better results.

In the followings, Section II describes the equivalent circuit battery model that is used for SOC and capacity estimation; Section III explains the mathematical analysis of the parameter identification, SOC estimation and the capacity estimation observer. Section IV presents simulation results to support the capacity estimation approach, and section V shows the results of applying the proposed identification and estimation approach to the experimental data; and Section VI concludes the paper.

II. BATTERY MODEL

The model for the battery can be as simple as a large capacitor with the capacity equal to the actual capacity of the
battery. Considering the resistance of the battery electrolyte regarding the moving of the ion carriers an internal resistance is added to the capacitance to build the terminal voltage. In addition to those steady state characteristics of the battery, the terminal voltage follows the step changes in the terminal current with a relaxation effect. This effect which shows the transient dynamic of the battery is modeled by several parallel Resistor-Capacitors (RC) pairs. The number of pairs is a trade-off between accuracy and simplicity of the model. Although in some references two pairs are suggested as the optimal number, one pair suffice representing the battery dynamics for electric vehicles in many references.

A. OCV-SOC relationship

Although in simple models for the battery the relationship between electromotive force (EMF) or Open Circuit Voltage (OCV) of the battery and the current is defined by the equations of a capacitor, the actual OCV follows a nonlinear relationship with the State of Charge (SOC) of the battery [10]. SOC is the amount of charge remained in the battery compared to the full capacity. The OCV-SOC relationship is a static characteristic of the battery, i.e. it does not change with the current and voltage profile of the battery. However, this curve significantly changes with temperature and ageing.

B. Battery State-Space Equivalent model

We model the battery dynamic with an RC equivalent that consists one RC pair as shown in figure 1. Also we use look-up table obtained from experimental data to represent the OCV-SOC relationship. Relying on the fact that the operating point’s moving on the OCV-SOC curve is usually very slow due to the large capacity of the battery compared to the normal C-rates, we consider a piecewise linear relationship between OCV and SOC at the operating point:

\[ V_{OC} = b_0 + b_1 SOC. \]  

(1)

With this assumption, system (2) represents the state space equations to describe the dynamic model of the battery. In these equations, the SOC of the battery and the voltage across the RC pair, \( V_{RC} \), are selected to be the system state variables.

\[
\begin{bmatrix}
\dot{SOC} \\
\dot{V}_{RC}
\end{bmatrix} = 
\begin{bmatrix}
0 & 0 \\
\frac{1}{R_{C}} & \frac{1}{V_{RC}}
\end{bmatrix}
\begin{bmatrix}
SOC \\
V_{RC}
\end{bmatrix} + 
\begin{bmatrix}
\frac{1}{Q_{R}} \\
\frac{1}{C}
\end{bmatrix}
i_L.
\]

\[ v_T = [b_1]
\begin{bmatrix}
SOC \\
V_{RC}
\end{bmatrix} + R_0i_L + b_0.
\]

(2)

Also, we assume that the terminal current (\( i_j \)) and voltage (\( v_T \)) are the only two values that are accessible from system (2). To obtain the estimated SOC as one of the states, the parameters in system (2) need to be identified. In our previous works we presupposed that we knew \( Q_R \) to be the nominal capacity of the battery and accordingly estimate \( \{b_0, R, C, R_0, b_1, S_{OC}, V_{RC}\} \) as \( \{\tilde{b}_0, \tilde{R}, \tilde{C}, \tilde{R}_0, \tilde{b}_1, \tilde{S}_{OC}, \tilde{V}_{RC}\} \) using system parameter identification methods and state estimation. But as explained, the ageing effect that could be calendar and/or cycling effect could degrade the actual capacity of the battery. In the next section, we will show that keeping the nominal capacity, \( Q_R \), instead of the actual capacity, \( Q_{act} \), in the equations of the battery will not degrade the identification results of the other parameters.

III. MATHEMATICAL ANALYSIS

A. Battery Parameter Identification

The parameters of the battery model that need to be estimated are \( \{b_0, R, C, R_0, b_1\} \). Since most of the parameter identification methods use the transfer function of the system to identify the parameters, first we obtain the transfer function of system (2):

\[ \frac{V(s)-b_0}{U(s)} = \frac{b_1}{s(1/R)} = \frac{b_0s^2 + b_1s + b_2}{s^2 + a_1s + a_2}. \]

(3)

From transfer function (3) and using bilinear transform \( s = \frac{2}{T} \frac{z-1}{z+1} \), we can get the discrete transfer function of system (2) with sample time \( T \):

\[ \frac{V(z^{-1})-b_0}{U(z^{-1})} = \frac{c_0+c_1z^{-1}+c_2z^{-2}}{1+a_1z^{-1}+a_2z^{-2}}. \]

(4)

In order to identify the parameters of a linear system like (4), the relationship between the system’s input/output (I/O) is described by a standard structure, such as the autoregressive exogenous model (ARX) model [11]:

\[ A(q)y(q) = B(q)u(q) + e(q), \]

(5)

where

\[ A(q) = 1 + a_1q^{-1} + \ldots + a_nq^{-n}, \]

(6)

\[ B(q) = b_0 + b_1q^{-1} + \ldots + b_mq^{-m}, \]

(7)

and \( e(q) \) is white noise (zero mean Gaussian noise). The LS identification approach provides a formula to minimize the Least Square (LS) error between this estimated output value and the real output at the present step. Since the I/O samples are being updated step-by-step while the system is running, a recursive least square (RLS) algorithm can be defined to identify the parameters of the system iteratively. Furthermore, because implementing the RLS algorithm is not easy in a real system and the I/O signal needs to be persistently exciting (PE) [7] at each step, we use the moving-window LS (MWLS) method, which is more practical. In this approach, the I/O data corresponding to a certain number (window) of past steps is used to estimate the
parameters. Identifying the coefficients of the discrete transfer function (4), the reverse bilinear transform \( z \rightarrow \frac{2+sz}{2-sz} \) is used to find the coefficients of the continuous-time transfer function (3). Therefore, assuming that the coefficients \( \{b_{00}, b_{11}, b_{22}, a_{11}, d_{22}\} \) have been identified correctly using the I/O data, we extract the battery parameters from the transfer function (3) coefficients as shown in equations 8-12.

\[
R_0 = b_{00} \quad \text{(8)}
\]

\[
RC = \frac{1}{a_{11}} \quad \text{(9)}
\]

\[
b_1 = \frac{Q_{\text{act}}RC b_{22}}{R_0} \quad \text{(10)}
\]

\[
\frac{1}{c} = b_{11} - \frac{R_0}{RC} - \frac{b_1}{Q_{\text{act}}} = b_{11} - \frac{R_0}{RC} - RC b_{22} \quad \text{(11)}
\]

\[
R = \frac{RC}{c} \quad \text{(12)}
\]

While \( R_0 \) and \( RC \) are not dependent on \( Q_{\text{act}} \) in equations (8) and (9), equation (10) shows that \( b_1 \) cannot be determined without an accurate approximation of \( Q_{\text{act}} \). Therefore, if there is a difference between \( Q_{\text{nom}} \) and \( Q_{\text{act}} \), the estimation of the \( b_1 \) will indicate the error. Nonetheless, when we use the non-accurate estimated \( b_1 \) to estimate \( C \) and \( R \), as demonstrated in equations (11) and (12), respectively, the \( Q_{\text{act}} \) is cancelled out and the estimated results do not depend on the \( Q_{\text{act}} \). To conclude, all the battery parameters except for \( b_1 \) can be identified uniquely without knowing the actual capacity of the battery. Since we use the OCV-SOC look-up table instead of the identified value of \( b_1 \) in SOC estimation algorithm, the estimated \( b_1 \) does not affect the estimation results.

\[A P + P^T A - P C^T R^{-1} CP = -Q,
\]  

(15)

Where \( Q \) and \( R \) are arbitrary semi-positive definite and positive definite matrices and the observer gain is obtained from equation (16),

\[
L^T = R^{-1} CP.
\]  

(16)

Figure 2 shows the block diagram that demonstrates the battery parameters/SOC co-estimation algorithm. All the battery parameters and the observer gain are being updated in the structure. As shown in this figure, we use the OCV-SOC function as a look-up table in the structure of the observer instead of using \( b_0 \) and \( b_1 \). Therefore, we use \( b_1 \) only in designing the observer gain. However, the change in the \( C \) matrix caused by different \( b_1 \) only affects the optimality of the designed observer from the convergence time and control effort point of view. Moreover, investigating the structure of the observer shows that matrix \( B \) also contains the capacity of the battery. If the nominal capacity, \( Q_{\text{nom}} \), is used instead of the actual capacity, \( Q_{\text{act}} \), to build this matrix, we can show that it does not influence the estimation of the SOC. That is because in this Luenberger type observer, the error between the actual state and the estimated state, \( e = \hat{x} - x \), the observer error, converges to zero or the observer is asymptotically stable if the matrix \( A-LC \) has all negative eigenvalues. Therefore, the convergence of the observer does not depend on the \( B \) matrix. Later on we will show that the simulation results endorse the fact that considering the nominal capacity instead of the actual capacity in the observer structure does not affect the SOC estimation results.

\[
\frac{1}{C}\begin{bmatrix}
\dot{x} = A x + Bu \\
y = Cx + Du + b_0
\end{bmatrix}
\]  

(13)

where \( x_1 = S_{oc} \), \( x_2 = V_{RC} \), \( A = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix} \), \( B = \begin{bmatrix} 1/Q_R \\ 1/C \end{bmatrix} \),

\[ C = \begin{bmatrix} b_1 & 1 \end{bmatrix}, D = R_0, u = I_L, y = V_{T}, x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \].

Therefore, the observer can be designed as a system with the equations (14):

\[
\begin{cases}
\dot{\hat{x}} = A \hat{x} + Bu + L(y - \hat{y}) \\
\hat{y} = C \hat{x} + Du + b_0
\end{cases}
\]  

(14)

where \( L^T = [L_1, L_2] \) is the observer gain vector. We use a linear quadratic (LQ) approach to design an optimal observer that minimizes the error and effort. In this method, the \( P \) matrix is calculated by solving the LQ Riccati equation (15),

\[
\frac{1}{Q_{\text{act}}} \begin{bmatrix}
\dot{x} = A x + Bu \\
y = Cx + Du + b_0
\end{bmatrix}
\]  

B. SOC Estimation

After identifying the parameters of the battery, an observer is designed to estimate the SOC, which is one of the states of the model. Assuming that the battery’s parameters \( \{R, C, R_0, b_1, b_0\} \) can be estimated as \( \{\hat{R}, \hat{C}, \hat{R}_0, \hat{b}_1, \hat{b}_0\} \), the battery model is represented as a system with equations (13):

\[
\begin{cases}
\dot{\hat{x}} = A \hat{x} + Bu \\
\hat{y} = C \hat{x} + Du + \hat{b}_0
\end{cases}
\]  

C. Design the battery capacity observer

After estimating the SOC with the parameters/SOC co-estimation method, we design another observer for a system that contains the coulomb counting equation to estimate the actual capacity of the battery. In this observer, we use this fact that the changes in the SOC of the battery ultimately follow the coulomb counting equation in which the capacity is the actual one:

\[
SOC = \frac{1}{Q_{\text{act}}} I.
\]  

(17)

We showed in the previous section that the estimation of SOC in our method is more based on the OCV of the battery rather than coulomb counting. Therefore, the result of SOC estimation can be used as the measured value to estimate the
actual capacity of the battery. We define the following system:

\[
\begin{aligned}
Q(k + 1) &= Q(k) + w(k) \\
SOC(k + 1) &= SOC(k) + \frac{1}{Q(k)} I_L , \\
y(k) &= SOC(k)
\end{aligned}
\]  

(18)

where \( Q(k) \) is the actual capacity of the battery and \( w(k) \) is a Gaussian Noise. Since one of the states of the system (18), SOC, can be observed directly from the output data, we design a reduced order observer (equation (19)) to estimate the capacity of the battery.

\[
\frac{1}{\hat{Q}(k+1)} = \frac{1}{Q(k)} + L (\hat{y}(k) - y(k)) ,
\]

(19)

where \( \hat{Q}(k) \) is the estimated capacity of the battery and \( \hat{y}(k) \) is the estimated output of system (18):

\[
\begin{aligned}
\hat{SOC}(k + 1) &= \hat{SOC}(k) + \frac{1}{\hat{Q}(k)} I_L \\
\hat{y}(k) &= \hat{SOC}(k)
\end{aligned}
\]

(20)

Since system (20) is nonlinear, instead of linear analytic design methods we use trial and error approach in this paper to design the observer gain, \( L \). Figure 3 shows the block diagram that implements the observer described by equation (19) to estimate the battery capacity. As previously explained, \( y(t) \) is the output for system (18) which is SOC of the battery.

![Fig. 3. The battery capacity observer](image)

IV. SIMULATION RESULTS

To demonstrate the robustness of the identification and SOC estimation results regarding the uncertainties in the full capacity calculation of the battery, we evaluate the results using the input/output data from a nonlinear model of the battery. In this model which has been developed in SIMULINK, we use a look-up table obtained from the experimental data to represent the OCV-SOC function. Also, the battery dynamics are represented by an RC equivalent circuit shown in figure 1 with fixed values for \( R_0, R \) and \( C \). Although those values change with SOC and C-rate in the real system, we keep them fixed in this model to make the verification easier. We obtain the current and voltage data of the model when the capacity of the battery is dropped by 20% of the nominal capacity, which is the extreme capacity degradation for most of the applications. It is similar to getting the current and voltage of a battery that has lost 20% of its capacity due to the cycling ageing effect. However, in the identification algorithm the nominal capacity is considered as the natural approximation of the full capacity of the battery. Applying the input/output data, demonstrated in figure 4, to the parameters identification algorithm, we compare the identified parameters to the results from an algorithm with the actual full capacity. The identification results for both nominal and actual capacities are demonstrated in figure 5 with a thicker line for nominal capacity to illuminate the difference. Also the dotted lines show the reference value for the parameters that have been used in the simulated model. The first three graphs show that \( R_0, R \) and RC are identified at the same values for both

![Fig. 4. Current and voltage data obtained from the battery model](image)

![Fig. 5. Comparison of the parameters identification results for the nominal and updated capacity](image)
nominal and actual capacities. These parameters are the major updating factors in the SOC co-estimation structure shown in figure 2. On the other side, as expected from equation 10, figure 5 shows that identification of $b_1$ is significantly affected by the assumption about the battery full capacity. However, following the earlier discussion, it does not influence the SOC estimation results because the experimental look-up table is used in the observer structure instead of $b_1$. The simulation results demonstrated in figure 6 confirms that considering the nominal capacity instead of the degraded one does not make any significant difference in the estimation of the SOC. On the contrary, in figure 6 the difference between the estimated SOCs with different full capacity considerations slightly increases for the SOCs between 30% and 60%. This is the area in figure 5 that the difference between the identified $b_1$s is minimum compared to other SOCs. Therefore, we can see again that lack of observability [9] has more negative influence on SOC estimation compared to the full capacity error influence. Afterwards, as shown in figure 7, when the estimated SOC is applied to the observer in figure 3 along with the battery current the parameters/SOC/capacity algorithm is able to accurately estimate the actual capacity of the battery.

![Fig. 6. Comparison of SOC estimations with nominal and updated capacity.](image)

![Fig. 7. Capacity estimation compared to the actual capacity of the battery.](image)

V. EXPERIMENTAL RESULTS

After verifying the performance of the parameters/SOC/capacity co-estimation algorithm using the simulated data, we apply the current and voltage data obtained from the experimental tests on 1.36 Ah lithium-polymer cells (Kokam SLPB723870H4) to estimate the actual capacity of the cells. In this test, we assume that the capacity of the brand new battery is equal to the nominal capacity. Therefore to evaluate the robustness of the algorithm, this time we assume that the full capacity of the battery in the parameters/SOC co-estimation algorithm is considered 20% lower than the nominal capacity. We again compare the results of the parameters/SOC co-estimation algorithm for both nominal and 20% degraded capacity in the algorithm structure. The identified parameters in figure 8 shows that even in the experimental case in which the parameters vary with SOC, the wrong assumption about the full capacity of the battery does not deviate the identification of the main parameters i.e. $R_0$, $R$ and $C$. This figure also shows that $b_1$ is the only parameter that is identified differently for different assumption about the full capacity. But we know that it does not affect the SOC estimation since look-up table is substituted in the algorithm structure. The SOC estimation results demonstrated in figure 9 confirms that considering a wrong value for the capacity does not influence the SOC estimation. Accordingly, we use the estimated SOC results to calculate the actual capacity of the lithium-Polymer battery cell using the online algorithm in figure 3. Figure 10 presents the capacity estimation results compared to the nominal capacity of the battery. This figure shows that although the initial value for the estimated capacity is zero the algorithm is able to compensate the initial state error and estimate the full capacity almost accurately before $t=2000s$. The increase in the capacity estimation error between 2000s and 4000s is due to the error in SOC estimation that roots in the observability issue.

![Fig. 8. Comparison of the parameters identification results for the actual and the degraded capacity on the experimental data.](image)
VI. CONCLUSION

Online estimation of the capacity is one of the most important characteristics of the battery. The capacity estimation is crucial to estimate the SOH and predict the end of life of the battery. Following our previous publications on developing an online adaptive parameters/ SOC co-estimation algorithm, we developed an algorithm that is able to estimate the actual capacity of the battery using the online SOC estimation results. To do so, we first showed with mathematical analysis that the parameters identification and SOC estimation results does not change with a wrong assumption about the full capacity of the battery. Later, the simulation and experimental results endorsed the analytical conclusions. After that we designed another observer that inputs the battery current and the estimated SOC and estimates the full capacity of the battery. We demonstrated with simulation and experimental data that the algorithm is able to estimate the actual capacity accurately when SOC estimation is accurate and the error occurs due to the weak observability.

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VII. REFERENCE


