Stabilization of networked PI control system using fuzzy logic modulation

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Abstract— **Despite of many advantages of using network in closed-loop control systems to do remote control and automation, the adverse effects such as performance degradation and system destabilization due to network-induced time delay are a major concern in networked control system design. In this paper, we will consider the problem of stabilizing the networked PI control system using fuzzy logic modulation. Using a version of the Hermite-Biehler Theorem applicable to quasipolynomials, a complete analytical characterizations of all stabilizing fuzzy logic modulator parameter values to stabilize the networked PI control system is provided. Numerical simulation of a network-based controlled DC motor is used to illustrate the proposed work.**

I. INTRODUCTION

In networked control systems (NCS), and due to the network-induced time delay effects, the available techniques used to control the NCS have to maintain the performance and stability of the system. Stability analysis of the NCS with network-induced time delay using the conventional state augmentation approach [1] is presented in [2, 3]. In [2] the authors used the hybrid systems stability analysis technique [4] to analyze the stability of the NCS with network-induced time delay. The problem of the stability in a first-order system with network-induced time delay is addressed in [5]. The effect of the network-induced time delay on the stability of a two-dimensional distributed system is analyzed in [6]. In [7], the stability condition for a time-variant uncertain discrete delay system is proposed. The effect of network-induced time delay on the stability of an autonomous mobile robot was investigated in [8, 9]. In [10, 11], the stability of the local NCS is proved if certain assumptions hold.

The characterization of the set of all stabilizing PID parameters using a version of the Hermite-Biehler Theorem [12] for the first-order plant with time delay is presented in [13, 14]. The solution for the PID stabilization problem is based on first determining the range of the proportional parameter for which a stabilizing PID controller exists. Then, for a fixed value of the proportional parameter in this range, it is shown that the set of stabilizing integral and derivative gain values lies in a convex polygon (either a trapezoid, a triangle, or a quadrilateral). A similar approach is applied for determining the stabilizing feedback gains for the second-order systems with time delay [15].

In this paper, we will consider the problem of stabilizing the networked PI control system using fuzzy logic modulator.

II. PROBLEM FORMULATION

A. *Networked PI control system*

In this section, a network-based controlled DC motor is used as an example to illustrate the use of Hermite-Biehler Theorem in providing the stability region of the networked PI control systems. The networked control system consists of three units: distributed remote unit (Remote controller + DC motor), central controller (PI controller), and communication network as shown in Fig. 1.

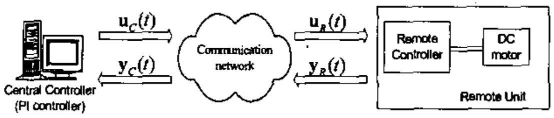

Figure 1. Networked PI control system.

A block diagram of the network-based controlled DC motor is shown in Fig. 2.

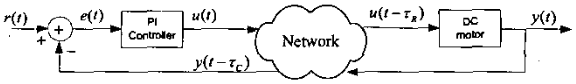

Figure 2. Block diagram of the network-based controlled DC motor.

The electro-mechanical dynamics of the DC motor can be described by the following first-order differential equations [16]:

$$\begin{aligned} u(t) = e_a &= L\frac{di_a}{dt} + R\,i_a + K_b\omega \\ J\frac{d\omega}{dt} + B\,\omega + T_l &= T_e = K\,i_a \end{aligned} \tag{1}$$

where $u = e_a$ is the armature winding input voltage; L is the armature winding inductance; i_a is the armature winding current; R is the armature winding resistance; J is the system moment of inertia; B is the system damping coefficient; K and K_b are the torque constant and the back emf constant, respectively; and ω is the rotor angular speed [16]. The remote controller in the remote unit is assumed to simply convert the control voltage signal from the central controller $u_C(t)$ into a PWM signal to drive the DC motor. The remote controller value $u_R(t)$ can be mathematically expressed as:

$$u_R(t) = u_C(t-\tau_R) \tag{2}$$

where τ_R is the delay time to transmit the control signal u_C

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Proceedings of the American Control Conference
Denver, Colorado June 4-6, 2003

from the central controller to the remote controller. The remote controller also sends the monitored signal $y_R(t)$ of the remote system back to the central controller, $y_C(t)$, and these two signals are related as:

$$y_C(t) = y_R(t - \tau_C) \tag{3}$$

where τ_C is the delay time to transmit the measured signal y_R from the remote controller to the central controller. The central controller uses a PI control algorithm [17] to compute the control signal required for the remote unit to track a step reference signal based on monitoring the system signals sent from the remote system via the network. The PI controller used has the form:

$$u(t) = K_p e(t) + K_i \int e(t)\,dt \tag{4}$$

where K_p and K_i are the proportional gain and integral gain of the PI controller, respectively; $e(t)=r(t)-y(t)$ is the error signal; $r(t)$ is the reference signal for the system to track, and $y(t)$ is the system output. In our case, $y(t) = \omega(t)$, the DC motor speed, and $u(t)$ is the input voltage to the DC motor.

B. Fuzzy modulation for networked PI control system

In this section, we will briefly present the fuzzy logic modulator technique to compensate for the network-induced time-delay effects in the networked PI control systems. This approach is based on modulating the control signal provided by the PI controller with a single parameter. The input to the fuzzy modulator is the error signal between the reference signal and the plant output signal; the output of the fuzzy modulator is the modulation parameter used to compensate for the effect of the network-induced time delay. The block diagram of the networked PI control system with fuzzy logic modulation applied to DC motor is shown in Fig. 3.

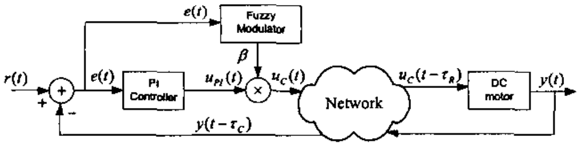


Figure 3. Network-based controlled DC motor with fuzzy modulation.

In Fig. 3, β is the modulation parameter that is the output of the fuzzy logic modulator system. The fuzzy logic modulator used in this paper is composed of the following two rules:

$$\begin{aligned} &\text{if } e \text{ is Small, then } \beta = \beta_1. \\ &\text{if } e \text{ is Large, then } \beta = \beta_2. \end{aligned} \tag{5}$$

such that $0 < \beta_1 < \beta_2 < 1$, where $\beta_i, i = 1,2$, are the consequent parameters corresponding to the modulation parameter β. For more on this compensation technique, please refer to our work in [18-20]. In [18, 19] we proposed an intelligent controller using fuzzy logic on top of a PI gain to adaptively compensate for the network-induced time delay in a time-delay sensitive networked control system application. Nonetheless, with this proposed fuzzy logic compensator, we will not completely redesign the existing PI controller; we will simply modulate the PI controller action with the output of the intelligent fuzzy logic controller to compensate for the network-induced time delay effects in the network-based control systems. A partial adaptation scheme for this fuzzy logic modulator is also proposed to tune the consequent parameters in the fuzzy rules. In [20] we proposed a full adaptive fuzzy modulation to compensate for the network-induced time delays in networked PI control systems. In this full AFM, not only are the consequent parameters be tuned adaptively, but also the membership functions parameters in the antecedent part of the fuzzy rules are tuned adaptively. In this paper, we will use the fuzzy logic modulation output parameter to stabilize the networked PI control system.

III. HERMITE-BIEHLER THEOREM APPLIED ON NETWORKED PI CONTROL SYSTEM

In this section, we will apply the version of Hermite-Biehler Theorem applicable to quasipolynomials to prove the stability of the networked PI control system. We will use the network-based controlled DC motor as an illustration example. The block diagram of the networked PI control system is shown in Fig.4

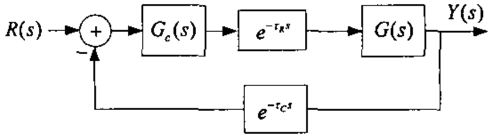


Figure 4. Block diagram of networked PI control system

For stability analysis purpose, and using block diagram reduction techniques, we can lump the sensor to controller time delay τ_C and the controller to actuator time delay τ_R together in L, where $L \ge (\tau_C + \tau_R)$ is the upper bound value on the network-induced time delays τ_C and τ_R as shown in Fig.5.

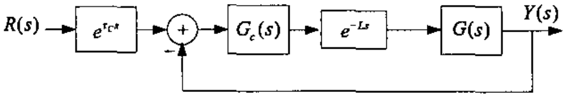


Figure 5. Simplified block diagram of networked PI control system.

In Fig. 5, the transfer function of the DC motor is:

$$G(s) = \frac{k}{(T_1 s+1)(T_2 s+1)} \tag{6}$$

where $k = 3.9804$ represents the dc gain of the DC motor, $T_1 = 4.4783 \times 10^{-4}$ and $T_2 = 0.0353$ represents the DC motor time constants, and

$$G_c(s) = \beta \frac{k_p s + k_i}{s} \tag{7}$$

is the PI controller transfer function with fuzzy modulator transfer function; k_p and k_i are the nominal values of PI controller parameters tuned without considering network-induced time delay effects. The characteristic equation of the system in Fig. 5 is given by

$$\begin{aligned}\Delta(s) &= s\left(T_1 s+1\right)\left(T_1 s+1\right)+k\beta\left(k_p s+k_i\right)e^{-Ls} \\ &= d(s)+e^{-Ls}n_1(s)\end{aligned} \tag{8}$$

Due to the presence of the exponential term e^{-Ls}, the number of roots of (8) is infinite, which makes such a stability check extremely difficult [13]. Instead of using (8), we can consider the quasipolynomial [15]

$$\begin{aligned}\Delta^{*}(s) &= e^{Ls}\Delta(s) \\ &= s\left(T_1 s+1\right)\left(T_1 s+1\right)e^{Ls}+k\beta\left(k_p s+k_i\right)\end{aligned} \tag{9}$$

Since e^{Ls} does not have any finite zeros, the zeros of $\Delta(s)$ are identical to those of $\Delta^{*}(s)$. We will use the extension of the Hermite-Biehler Theorem [12, 14] to check the stability of $\Delta(s)$ and thus the stability of the networked PI control system.

Theorem 1: Let $\Delta^{*}(s)$ be given by (9), and write $\Delta^{*}(j\omega)=\delta_r(\omega)+j\delta_i(\omega)$ where $\delta_r(\omega)$ and $\delta_i(\omega)$ represent respectively the real and imaginary parts of $\Delta^{*}(j\omega)$. Under the following assumptions:

$A1$. $\deg\left[d(s)\right]=n$ and $\deg\left[n_1(s)\right]\leq n$.

$A2$. $L>0$.

$\Delta^{*}(s)$ is stable if and only if : 1) $\delta_r(j\omega)$ and $\delta_i(j\omega)$ have only simple real roots and these interlace, 2) $E(\omega_o)=\delta_i^{'}(j\omega_o)\delta_r(j\omega_o)-\delta_i(j\omega_o)\delta_r^{'}(j\omega_o)>0$, for some $\omega_o\in(-\infty,\infty)$; where $\delta_i^{'}(j\omega_o)$ and $\delta_r^{'}(j\omega_o)$ denote the first derivative with respect to ω of $\delta_i(j\omega_o)$ and $\delta_r(j\omega_o)$, respectively.

First, we will start testing the stability of the networked PI control system without including the network-induced time delays, i.e., L=0. The characteristic equation of the delay-free DC motor with PI controller is

$$\Delta(s)=s\left(T_1 s+1\right)\left(T_2 s+1\right)+kk_p s+kk_i \tag{10}$$

Using Routh stability criterion [21], the following conditions on the PI controller parameters k_p and k_i are required for a delay-free stable DC motor operation:

$$k_i>0, \text{and } k_p>0 \tag{11}$$

Since the minimal requirement for any control design is that the delay-free closed-loop system be stable, the inequalities in (11) are assumed hold through this paper.

Now, we will test the stability of the networked PI control system in the presence of network-induced time delays. For our network-based controlled DC motor, the assumptions used in Theorem 1 to check the stability of $\Delta^{*}(s)$ are satisfied since $\deg\left[d(s)\right]=3$ and $\deg\left[n_1(s)\right]=1<3$, and from the nature of the network, the network-induced time delay is positive. After several simple substitutions, and with the change of variables $z=\omega L$, the real and imaginary parts of $\Delta^{*}(j\omega)$ can be rewritten as:

$$\delta_r(z)=\tfrac{z}{L}\left(\tfrac{z^2}{L^2}T_1T_2-1\right)\sin z-\tfrac{z^2}{L^2}\left(T_1+T_2\right)\cos z+k\beta k_i \tag{12}$$

$$\delta_i(z)=\tfrac{z}{L}\left(1-\tfrac{z^2}{L^2}T_1T_2\right)\cos z-\tfrac{z^2}{L^2}\left(T_1+T_2\right)\sin z+\tfrac{z}{L}k\beta k_p \tag{13}$$

With simple check, we can verify that $\delta_r(z)$ is an even function in z, while $\delta_i(z)$ is an odd function in z. From Theorem 1, we need to check two conditions to insure the stability of the quasipolynomial $\Delta^{*}(s)$:

A. Condition 2 in Theorem 1

For $\omega_o=z_o=0$, condition 2 becomes:

$$\tfrac{1}{L}\left(1+k\beta k_p\right)k\beta k_i>0 \tag{14}$$

which is satisfied since $L>0, k>0, \left(k_p,k_i\right)>0, \text{and } \beta>0$.

B. Condition 1 in Theorem 1

From (13) we can compute the roots of the imaginary part, i.e., $\delta_i(z)=0$. This gives us the following equation:

$$\tfrac{z}{L}\left[\left(1-\tfrac{z^2}{L^2}T_1T_2\right)\cos z-\tfrac{z}{L}\left(T_1+T_2\right)\sin z+k\beta k_p\right]=0 \tag{15}$$

Then either $z=0$, or

$$\left(1-\tfrac{z^2}{L^2}T_1T_2\right)\cos z-\tfrac{z}{L}\left(T_1+T_2\right)\sin z+k\beta k_p=0 \tag{16}$$

From this it is clear that one root of the imaginary part is $z_{\omega}=0$. The other roots are difficult to find since we need to solve (16) analytically. However, we can plot the terms involved in (16) and graphically examine the nature of the solution [14].

For illustration, let L=8 (msec), and $\beta=0.26$, a plot of the terms $\left(1-\tfrac{z^2}{L^2}T_1T_2\right)\cos z+k\beta k_p$ and $\tfrac{z}{L}\left(T_1+T_2\right)\sin z$ is shown in Fig. 6.

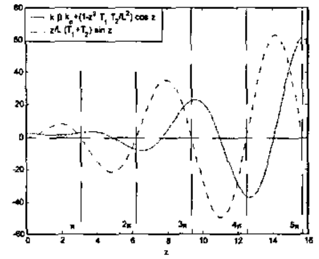


Figure 6. Plot of $\left(1-\dfrac{z^2}{L^2}T_1T_2\right)\cos z+k\beta k_p$ and $\dfrac{z}{L}\left(T_1+T_2\right)\sin z$.

In order to check if $\delta_i(z)$ has only real roots; we need to use the following Theorem [14].

Theorem 2: Let M and N denote the highest powers of s and e^{s} respectively in $\Delta^{*}(s)$. Let η be an appropriate constant such that the coefficients of terms of highest degree in $\delta_r(\omega)$ and $\delta_i(\omega)$ do not vanish at $\omega=\eta$. Then for equations $\delta_r(\omega) = 0$ or $\delta_i(\omega) = 0$ to have only real roots, it is necessary and sufficient that in the intervals

$$-2l\pi + \eta \le \omega \le 2l\pi + \eta, \qquad l = 1, 2, 3, \ldots$$

$\delta_r(\omega)$ or $\delta_i(\omega)$ have exactly $4lN+M$ real roots starting with a sufficient large l.

Substituting $s_1 = Ls\cdot$ in the expression for $\Delta^*(s)$, we see that for the new quasipolynomial in s_1, M=3 and N=1. Next we choose $\eta = \frac{\pi}{15} = 0.2094$ to satisfy the requirement such that the coefficients of terms of highest degree in $\delta_r(z)$ and $\delta_i(z)$ do not vanish at $z = \eta$. From Fig. 6, $\delta_i(z)$ has $4N$+M=7 real roots in the interval $\left[-2\pi + \frac{\pi}{15}, 2\pi + \frac{\pi}{15}\right]$, and it is clear that $\delta_i(z)$ has two real roots in each of the intervals $\left[2l\pi + \frac{\pi}{15}, 2(l+1)\pi + \frac{\pi}{15}\right]$ and $\left[-2(l+1)\pi + \frac{\pi}{15}, -2l\pi + \frac{\pi}{15}\right]$ for l=1,2... Hence, it follows that $\delta_i(z)$ has exactly $4lN+M$ in the interval $\left[-2l\pi + \frac{\pi}{15}, 2l\pi + \frac{\pi}{15}\right]$. From Theorem 2, we conclude that $\delta_i(z)$ has only real roots.

Now we will evaluate $\delta_r(z)$ at the roots of the imaginary part $\delta_i(z)$. For z_{io}=0, and using (12) we obtained $\delta_r(z_{io})$=43.8, for other $\delta_i(z)$ roots, we get:

$\delta_r(z_{i1}) = -23.2, \delta_r(z_{i2}) = 4848, \delta_r(z_{i3}) = -21234, \delta_r(z_{i4}) = 49414,$
$\delta_r(z_{i5}) = -95876, \delta_r(z_{i6}) = 162130$

The interlacing of roots of $\delta_r(z)$ and $\delta_i(z)$ is satisfied which is equivalent to:

$$\delta_r(z_{io}) > 0, \delta_r(z_{i1}) < 0, \delta_r(z_{i2}) > 0, \ldots \tag{17}$$

From the interlacing property of $\delta_i(z)$ and $\delta_r(z)$, and the fact that $\delta_i(z)$ has only real roots, we conclude that $\delta_r(z)$ also has only real roots. Thus all conditions of Theorem 1 are satisfied and the quasipolynomial Δ^*(s) is stable for the given β and L values, i.e., the networked PI control system with fuzzy logic modulation is stable for the given β and L values.

IV. STABILIZATION USING FUZZY LOGIC MODULATOR PARAMETER

Now, we will consider the problem of stabilizing the networked PI control system using fuzzy logic modulator. Using a version of the Hermite-Biehler Theorem applicable to quasipolynomials, a complete analytical characterization of all stabilizing fuzzy logic modulator parameter values to stabilize the networked PI control systems is provided. In this theorem we assume a small network-induced time delay variation.

Theorem 3: The range of the fuzzy logic modulator parameter β for which the networked PI control system can be stabilized using fuzzy logic modulator is given by

$$0 < \beta < \frac{1}{kk_p}\left[\left(T_1 + T_2\right)\frac{\alpha}{L}\sin\alpha - \left(1 - \frac{\alpha^2}{L^2}T_1T_2\right)\cos\alpha\right]$$

such that the interlacing condition on the roots of $\delta_i(z)$ and $\delta_r(z)$ is satisfied, where α is the solution of the equation:

$$\tan\alpha = -\frac{\left(\frac{T_1+T_2}{L}\right)+\left(2\frac{T_1T_2}{L^2}\right)}{\left(\frac{T_1+T_2}{L}\right)+\left(1-\frac{\alpha^2T_1T_2}{L^2}\right)}\alpha$$

Proof:

First, from the definition of fuzzy logic modulation parameter in section II-B, this implies that $\beta > 0$. Then, we need to show that the imaginary part of $\Delta^*(j\omega)$ has only simple real roots if and only if

$$\beta < \frac{1}{kk_p}\left[\left(T_1 + T_2\right)\frac{\alpha}{L}\sin\alpha - \left(1 - \frac{\alpha^2}{L^2}T_1T_2\right)\cos\alpha\right] \tag{18}$$

Rewrite (16) as $\frac{\left(1-\frac{z^2}{L^2}T_1T_2\right)\cos z+k\beta k_p}{\sin z} = \left(T_1 + T_2\right)\frac{z}{L}$, we can define β_u as the largest number of β so that the plot of $\frac{\left(1-\frac{z^2}{L^2}T_1T_2\right)\cos z+k\beta k_p}{\sin z}$ intersects the line $\left(T_1 + T_2\right)\frac{z}{L}$ twice in the interval $(0, \pi)$. For the case $\beta > \beta_u$ and using Theorem 2, one can easily argue that in this case all roots of $\delta_i(z)$ will not be real, thereby ruling out closed-loop stability.

From the definition of β_u; it follows that if $\beta = \beta_u$ the plot of $\frac{\left(1-\frac{z^2}{L^2}T_1T_2\right)\cos z+k\beta k_p}{\sin z}$ intersects the line $\left(T_1 + T_2\right)\frac{z}{L}$ only once in the interval $(0, \pi)$. Let α denote the value of z for which this intersection occurs. Then for $z = \alpha \in (0, \pi)$ we have

$$\frac{\left(1-\frac{\alpha^2}{L^2}T_1T_2\right)\cos\alpha+k\beta_u k_p}{\sin\alpha} = \left(T_1 + T_2\right)\frac{\alpha}{L} \tag{19}$$

Rewrite (19),

$$\left(1 - \frac{\alpha^2}{L^2}T_1T_2\right)\cos\alpha + k\beta_u k_p = \left(T_1 + T_2\right)\frac{\alpha}{L}\sin\alpha \tag{20}$$

Moreover, at $z = \alpha$, the line $\left(T_1 + T_2\right)\frac{z}{L}$ is tangent to the plot of $\frac{\left(1-\frac{z^2}{L^2}T_1T_2\right)\cos z+k\beta_u k_p}{\sin z}$. Thus

$$\frac{d}{dz}\left[\frac{\left(1-\frac{z^2}{L^2}T_1T_2\right)\cos z+k\beta k_p}{\sin z}\right]_{z=\alpha} = \frac{(T_1+T_2)}{L} \tag{21}$$

After few lines of derivations,

$$\frac{-\left(1-\frac{\alpha^2}{L^2}T_1T_2\right)-2\frac{T_1T_2}{L^2}z\sin z\cos z-k\beta_u k_p\cos z}{\sin^2\alpha} = \frac{(T_1+T_2)}{L} \tag{22}$$

Rewrite (22),

$$\begin{aligned}&-\left(1 - \frac{\alpha^2}{L^2}T_1T_2\right) - 2\frac{T_1T_2}{L^2}\alpha\sin\alpha\cos\alpha - k\beta_u k_p\cos\alpha \\ &= \frac{(T_1+T_2)}{L}\sin^2\alpha\end{aligned} \tag{23}$$

Eliminating $k\beta_u k_p$ between equations (20), (23) by multiplying (20) with $\cos\alpha$ and add to it (23). After a few simple derivations, we conclude that $\alpha \in (0,\pi)$ can be obtained as a solution of the following equation:

$$\tan\alpha = -\frac{\left(\frac{T_1+T_2}{L}\right)+\left(2\frac{T_1T_2}{L^2}\right)}{\left(\frac{T_1+T_2}{L}\right)+\left(1-\frac{\alpha^2 T_1T_2}{L^2}\right)}\alpha \tag{24}$$

Once α is determined, the parameter β_u can be obtained using (20):

$$\beta_u = \frac{1}{kk_p}\left[\left(T_1+T_2\right)\frac{\alpha}{L}\sin\alpha-\left(1-\frac{\alpha^2}{L^2}T_1T_2\right)\cos\alpha\right] \tag{25}$$

In order to check the interlacing condition on roots of $\delta_i(z)$ and $\delta_r(z)$, equation (17) is used.

For the fuzzy logic modulator parameter β in this range, i.e., $\delta_i(z)$ has only real roots, and from the satisfaction of interlacing condition of roots $\delta_i(z)$ and $\delta_r(z)$, we conclude that $\delta_r(z)$ also has only real roots. Thus all conditions of Theorem 1 are satisfied and the quasipolynomial $\Delta^{*}(s)$ is guarantee stable by the given fuzzy logic modulator parameter β, i.e., the networked PI control system is stabilized by the fuzzy logic modulator parameter. This completes the proof of the Theorem 3. In the following sections, we will highlight some remarks on the stability of the networked PI control system.

A. *Increasing in stability region*

With the fuzzy logic modulation, we increased the stability region of the network-based controlled DC motor. Fig. 7 shows that with the nominal PI controller (i.e., $\beta = 1$), it can stabilize the networked PI control system up to L= 9 (msec), while for the network-induced time delay larger than this upper bound value, the nominal PI controller cannot stabilize the networked PI control system. On the other hand, with the fuzzy logic modulator, the stability region is increased as shown in Fig. 7.

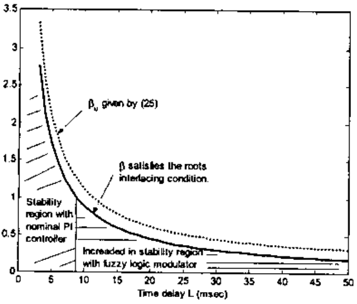


Figure 7. Plot of the stabilizing fuzzy logic modulator β as a function of L.

B. *Delay-free closed loop system stability*

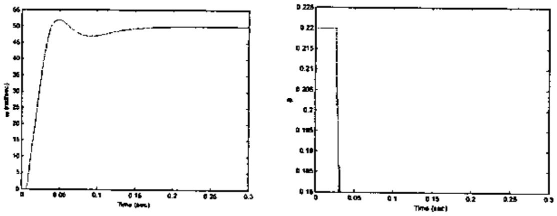
With L=0, i.e., no time delay, equation (24) reduced to $\tan\alpha = \frac{2}{\alpha}$, which can be solved analytically to obtain $\alpha = 1.08$. Substituting in (25), $\beta_u = \infty$ which is consistent the result in (11), i.e., both k_p and k_i are less than ∞.

V. SIMULATION RESULTS AND DISCUSSION

In this section we will simulate the dynamic responses of the network-based controlled DC motor corresponding to stable region, unstable region, and on border of stable/unstable regions shown in Fig. 7.

A. *Stable region*

In this region, and based on Theorem 3, the fuzzy logic modulator parameter should be less than its upper bound value provided by Theorem 3. The typical step response of the network-based controlled DC motor subject to tracking a reference signal of 50 (rad/sec) is shown in Fig. 8(a). The corresponding fuzzy logic modulator satisfies Theorem 3 (where the upper bound value of the fuzzy logic modulator is 0.59 corresponding to L=15 (msec)) is shown in Fig. 8(b).



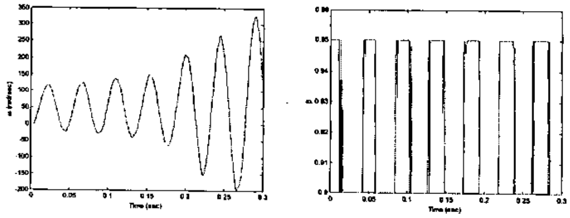
(a) (b)

Figure 8. (a) Typical step response of the network-based controlled DC motor, (b) Corresponding fuzzy logic modulator parameter β.

Thus, by having the fuzzy logic modulator parameter less than the upper bound value provided by Theorem 3, we guarantee the stability of the networked PI control system with fuzzy logic modulator.

B. *Unstable region*

In this region, the fuzzy logic modulator parameter is greater than its upper bound value provided by Theorem 3. The typical step response of the network-based controlled DC motor subject to tracking a reference signal of 50 (rad/sec) is shown in Fig. 9(a). The corresponding fuzzy logic modulator, which violates Theorem 3 (where the upper bound value of the fuzzy logic modulator is 0.89 corresponding to L=10 (msec)) is shown in Fig. 9(b).



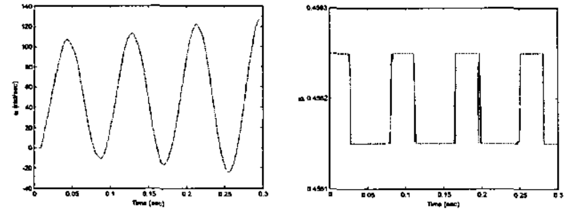
(a) (b)

Figure 9. (a) Typical step response of the network-based controlled DC motor, (b) Corresponding fuzzy logic modulator parameter β.

The simulation results in Fig. 9 suggest that during the adaptation process of the adaptive fuzzy logic modulation, the upper bound of the fuzzy logic modulator provided by Theorem 3 should be taken into consideration; such that we adapt the fuzzy logic modulator and at the same time stabilize the networked PI control system.

C. On border of stable/unstable regions

In the boundary of stable/unstable region, the fuzzy logic modulator parameter is equal to its upper bound value provided by Theorem 3. The typical step response of the network-based controlled DC motor subject to tracking a reference signal of 50 (rad/sec) is shown in Fig. 10(a). The corresponding fuzzy logic modulator equals to the upper bound value on the fuzzy logic modulator of 0.4562 (corresponding to L=20 (msec)) is shown in Fig. 10(b).



(a) (b)

Figure 10. (a) Typical step response of the network-based controlled DC motor, (b) Corresponding fuzzy logic modulator parameter β .

As shown in Fig. 10, the networked PI control system has unstable oscillatory behavior since the fuzzy logic modulator parameter is equals to the upper bound value provided by Theorem 3.

VI. CONCLUSION

In this paper, we considered the problem of stabilizing the networked PI control system using fuzzy logic modulator. Using a version of the Hermite-Biehler Theorem applicable to quasipolynomials, a complete analytical characterization of all stabilizing fuzzy logic modulator parameter values to stabilize the networked PI control systems is provided. Numerical simulation of a network-based controlled DC motor is used to illustrate the proposed work.

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