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Technical Notes

Flux-Depression Factors for Scattering and Absorbing Media

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ABSTRACT

The method of elementary solutions, along with Chandrasekhar's invariance principles, is used to solve a two-region problem relevant to flux-depression calculations. An improved P-L-type approximation is also discussed.

INTRODUCTION

In a recent Note,[1] the elementary solutions[2] of the one-speed transport equation and the invariance principles of Chandrasekhar[3] were used to solve concisely and accurately the critical problem for a reflected reactor. Here we develop, in a similar manner, the solution to a two-region problem basic to flux-depression factors for, say, foil-activation or control-rod worth calculations.

We consider the one-speed transport equations for region 1, $-a \leq x \leq a$, and region 2, $|x| > a$, written in the familiar[4] manner

$$\mu \frac{\partial}{\partial x} \Psi_\alpha(x,\mu) + \Psi_\alpha(x,\mu) = \frac{1}{2} c_\alpha \int_{-1}^{1} \Psi_\alpha(x,\mu')d\mu' + \delta_{\alpha,2} \quad , \tag{1}$$

where $\delta_{\alpha,2}$ is used to denote the presence of a constant source throughout region 2. Here, $c_1 < 1$ and $c_2 < 1$, and thus we seek bounded solutions of Eq. (1) such that $\Psi_\alpha(-x,-\mu) = \Psi_\alpha(x,\mu)$ and $\Psi_1(a,\mu) = \Psi_2(a,\mu)$, $\mu \in (-1, 1)$. We note that Williams[5] has summarized several basic contributions to the solution of this problem and has given analytical and numerical results valid for essentially absorbing thin foils.

ANALYSIS

For region 1, we can write the angular flux as

$$\Psi_1(x,\mu) = A(\nu_0)[\phi_1(\nu_0,\mu) \exp(-x/\nu_0) + \phi_1(-\nu_0,\mu) \exp(x/\nu_0)] + \int_0^1 A(\nu)[\phi_1(\nu,\mu) \exp(-x/\nu) + \phi_1(-\nu,\mu) \exp(x/\nu)]d\nu \quad , \tag{2}$$

where we have used an established notation for Case and Zweifel's elementary solutions.[4] For region 2, we write

$$\Psi_2(x,\mu) = B(\eta_0)\phi_2(\eta_0,\mu) \exp(-x/\eta_0) + \int_0^1 B(\eta)\phi_2(\eta,\mu) \exp(-x/\eta)d\eta + \frac{1}{1 - c_2} \quad , \quad x > a \quad , \tag{3}$$

where we have added a particular solution to account for the source term. Instead of the conventional continuity condition at $x = a$, i.e., $\Psi_1(a,\mu) = \Psi_2(a,\mu)$, $\mu \in (-1, 1)$, we prefer to use Chandrasekhar's S function,

$$S_2(\mu',\mu) = \frac{c_2\mu'\mu}{\mu' + \mu} H_2(\mu')H_2(\mu) \quad , \tag{4}$$

to deduce the boundary condition

$$\Psi_1(a,-\mu) = \frac{1}{2\mu} \int_0^1 S_2(\mu',\mu)\Psi_1(a,\mu')d\mu' + \frac{1}{(1 - c_2)^{1/2}} H_2(\mu) \quad , \quad \mu \in (0, 1) \ . \tag{5}$$

Here $H_2(\mu)$ is the usual[3] H function for region 2. We note that once $\Psi_1(x,\mu)$ is constrained to satisfy Eq. (5), we can readily deduce the coefficients $B(\eta_0)$ and $B(\eta)$ required to establish $\Psi_2(x,\mu)$.

On substituting Eq. (2) into Eq. (5) and performing the indicated integration over μ', we find

$$\frac{A(\nu_0)}{H_2(\nu_0)} \exp(a/\nu_0)\phi_1(\nu_0,\mu) + \int_0^1 \frac{A(\nu)}{H_2(\nu)} \exp(a/\nu)\phi_1(\nu,\mu)d\nu = A(\nu_0)\left(\frac{c_2}{c_1} - 1\right) H_2(\nu_0) \exp(-a/\nu_0)\phi_1(-\nu_0,\mu) + \int_0^1 A(\nu)\left(\frac{c_2}{c_1} - 1\right) H_2(\nu) \exp(-a/\nu)\phi_1(-\nu,\mu)d\nu + (1 - c_2)^{-1/2} \quad , \quad \mu > 0 \quad , \tag{6}$$

which can readily be regularized, as discussed previously,[1] to yield

$$E(\nu_0) = \frac{2}{H(\nu_0)} [\exp(2z_{0,M}/\nu_0) + \exp(-2a/\nu_0)]^{-1} \times \left[\frac{2}{(c_1 - c_2)}\left(\frac{1 - c_1}{1 - c_2}\right)^{1/2} - \int_0^1 \frac{E(\nu')H(\nu')\nu'}{\nu' + \nu_0} \exp(-2a/\nu')d\nu'\right] \tag{7}$$

and

[1] C. E. SIEWERT and A. R. BURKART, *Nucl. Sci. Eng.*, **58**, 253 (1975).
[2] K. M. CASE, *Ann. Phys.*, **9**, 1 (1960).
[3] S. CHANDRASEKHAR, *Radiative Transfer*, Oxford University Press, New York and London (1950).
[4] K. M. CASE and P. F. ZWEIFEL, *Linear Transport Theory*, Addison-Wesley Publishing Co., Inc., Reading, Massachusetts (1967).
[5] M. M. R. WILLIAMS, *Proc. Phys. Soc.*, **85**, 413 (1965).