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Technical Notes

An Improved *P*-*L* Solution to the Reflected Critical-Reactor Problem in Slab Geometry

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ABSTRACT

Principles of invariance are used with the traditional P-L method to yield concise and improved results for critical calculations of reflected reactors.

ANALYSIS

The purpose of this Note is to report the results of a simple approximate solution to the reflected critical-slab problem. The technique blends the features of the Chandrasekhar *H* function[1] with the usual *P*-*L* approximation to give a result for the critical dimension that is consistently more accurate and, in a sense, less complicated than that obtained from the traditional *P*-*L* method.

We seek a solution to

$$\mu \frac{\partial}{\partial x} \psi_\alpha(x,\mu) + \psi_\alpha(x,\mu) = \tfrac{1}{2} c_\alpha \int_{-1}^{1} (1 + b_\alpha \mu\mu')\psi_\alpha(x,\mu')\,d\mu' \quad , \tag{1}$$

where $\psi_1(x,\mu)$ denotes the angular flux in the core, $x \epsilon (-a, a)$, and where $\psi_2(x,\mu)$ is the angular flux in the infinite reflector, $|x| > a$. Here we consider that c_1, c_2, b_1, and b_2 are given, and thus we seek the value of a for which there exists a real non-negative solution of Eq. (1), subject to the boundary conditions

$$\psi_\alpha(x,\mu) = \psi_\alpha(-x,-\mu) \quad , \tag{2a}$$

$$\lim_{|x|\to\infty} \psi_2(x,\mu) = 0 \quad , \tag{2b}$$

and

$$\psi_1(a,\mu) = \psi_2(a,\mu) \quad , \quad \mu \epsilon (-1,1) \quad . \tag{2c}$$

We note that a rigorous solution of this problem is reported in a related paper[2]; but here, we wish to give a simple approximate solution that can be useful for low-order calculations. For the core, we can write the properly symmetric *P*-*L* solution, for *L* odd, as

$$\psi_1(x,\mu) = \sum_{l=0}^{L} \sum_{j=1}^{[(L+1)/2]} \left(\frac{2l+1}{2}\right) P_l(\mu) T_l(\nu_j) A_j \times [\exp(-x/\nu_j) + (-1)^l \exp(x/\nu_j)] \quad , \tag{3}$$

where $P_l(\mu)$ denotes the Legendre polynomial, the arbitrary coefficients A_j are to be determined from the boundary conditions, and the polynomials $T_l(\xi)$ follow from[1]

$$[2l + 1 - c_1(\delta_{l,0} + b_1 \delta_{l,1})]\,\xi T_l(\xi) = (l+1) T_{l+1}(\xi) + l T_{l-1}(\xi) \quad , \tag{4}$$

with $T_0(\xi) = 1$. Also, the eigenvalues ν_j required in Eq. (3) are the "positive" $\frac{1}{2}(L+1)$ zeros of $T_{L+1}(\xi)$.

Were we to pursue the traditional *P*-*L* approximation, we would now write an expression similar to Eq. (3) for the reflector, equate moments of the two expressions evaluated at $x = a$, and thus obtain a critical condition that the determinant of the resulting $(L+1) \times (L+1)$ coefficient matrix must vanish. Here, however, since $c_2 < 1$, we can use the Chandrasekhar result,[1]

$$\psi_2(a,-\mu) = \frac{1}{2\mu} \int_0^1 S(\mu',\mu)\psi_2(a,\mu')\,d\mu' \quad , \quad \mu \epsilon (0,1) \quad , \tag{5}$$

where

$$S(\mu',\mu) = \frac{c_2 \mu\mu'}{\mu+\mu'} H(\mu') H(\mu) [1 - \hat{c}(\mu+\mu') - l_2 \mu\mu'] \quad , \tag{6}$$

with

$$l_2 = b_2(1 - c_2) \quad , \quad \hat{c} = \frac{c_2 l_2 H_1}{2 - c_2 H_0} \quad , \quad \text{and} \quad H_\alpha = \int_0^1 \mu^\alpha H(\mu)\,d\mu \quad , \tag{7}$$

to write the continuity condition, Eq. (2c), as

$$\psi_1(a,-\mu) = \frac{1}{2\mu} \int_0^1 S(\mu',\mu)\psi_1(a,\mu')\,d\mu' \quad , \quad \mu \epsilon (0,1) \quad . \tag{8}$$

We note that $H(\mu)$ required in the foregoing is the Chandrasekhar *H* function.[1] In addition to an analytical expression for $H(\mu)$, there exists the useful (for numerical calculations) nonlinear integral equation,

$$\frac{1}{H(\mu)} = 1 - \frac{c_2}{2} \mu \int_0^1 (1 + l_2 x^2) H(x) \frac{dx}{x+\mu} \quad , \quad \mu \epsilon [0,1] \quad . \tag{9}$$

If we now enter Eq. (3) into Eq. (8), we can use Eq. (9) and the recursive relation,

$$(2l+1)\mu P_l(\mu) = (l+1) P_{l+1}(\mu) + l P_{l-1}(\mu) \quad , \tag{10}$$

to evaluate the integration over μ' to obtain

$$H(\mu) \sum_{l=0}^{L} \sum_{j=1}^{[(L+1)/2]} \left(\frac{2l+1}{2}\right) \pi_l(-\mu) T_l(\nu_j) A_j \times [\exp(-a/\nu_j) + (-1)^l \exp(a/\nu_j)] = 0 \quad , \quad \mu \epsilon (0,1) \quad . \tag{11}$$

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[1] S. CHANDRASEKHAR, *Radiative Transfer*, Oxford University Press, London and New York (1950).

[2] A. R. BURKART, Y. ISHIGURO, and C. E. SIEWERT, *Nucl. Sci. Eng.*, **61**, 72 (1976).