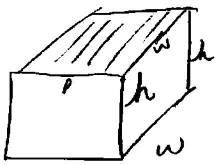
Solutions

- A studio builds a display box: (etc)



$$Cost = C = 20(2lw) + 12(2lh + 2wh)$$

Constraints: $l = w$

$$250 = lwh \Rightarrow 250 = l^2 h \Rightarrow h = \frac{250}{l^2}$$

So $C = 40l^2 + 24lh + 24lh = 40l^2 + 48lh$

$$C = 40l^2 + 48l\frac{250}{l^2} = 40l^2 + \frac{12000}{l}$$

$$C' = 80l - \frac{12000}{l^2} = 0$$

$$80l = \frac{12000}{l^2}$$

$$80l^3 = 12000$$

$$l^3 = 125$$

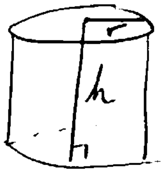
$$l = 5$$

Check: $C'' = 80 + \frac{24000}{l^3} > 0$ when $l=5$

so l is a min.

$l=5 \Rightarrow w=5 \Rightarrow h = \frac{250}{25} = 10$ $\quad 5\times5\times10$

- A cylindrical can is to be made to hold $1L \doteq 1000cm^3$ of oil. Find the dimensions that will minimize the cost of the can.



$$SA = 2\pi r^2 + 2\pi rh$$

Constraint

$$1000 = \pi r^2 h \quad \text{so } h = \frac{1000}{\pi r^2}$$

$$SA = 2\pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right) = 2\pi r^2 + \frac{2000}{r}$$

$$SA' = 4\pi r - \frac{2000}{r^2} = \frac{4(\pi r^3 - 500)}{r^2} = 0$$ when numerator is zero.

$\pi r^3 - 500 = 0$ or $\pi r^3 = 500$

$$r = \sqrt[3]{\frac{500}{\pi}} \approx 5.419\,cm$$

check $SA'' = 4\pi + \frac{4000}{r^3} > 0$

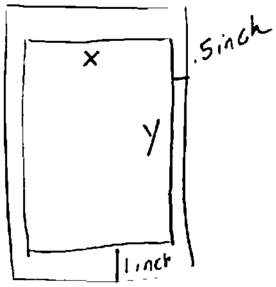
so $\cup$

so $r = 5.419$ is a min

$$h = \frac{1000}{\pi\left(\sqrt[3]{\frac{500}{\pi}}\right)^2} \text{ or } 2r \approx 10.838\,cm$$

when simplified

• A paper has one-inch margins on top & bottom and half-inch margins on the sides. You want a printable area of 50 sq. inches, Find the dimensions that minimize the amount of paper needed.



Paper used = $A = (y+2)(x+1)$

Constraint:

$$50 = xy \quad \text{or} \quad y = \frac{50}{x}$$

$$A = \left(\frac{50}{x}+2\right)(x+1) = 50 + \frac{50}{x} + 2x + 2$$

$$A' = -\frac{50}{x^2} + 2 = 0 \quad \text{when}$$

$$-\frac{50}{x^2} + 2 = 0 \Rightarrow 2 = \frac{50}{x^2}$$

$$\Rightarrow 2x^2 = 50$$

$$x^2 = 25$$

$$x = 5$$

check

$A'' = \frac{50}{x^3} > 0$ when $x = 5$, $\cup$

$x = 5$ is a min

$$y = \frac{50}{x} = \frac{50}{5} = 10$$

$5^{in} \times 10^{in}$