MA 241 Section 6.5 Supplement

Units

	Force	Distance	Work	Weight of Water
Customary	lb	ft	ft-lb	62.5 lb/ft^3
SI	N = kg $\cdot$ m/s^2	m	J	9800 N/m^3

Note: In SI, we include acceleration due to gravity, so $F = 9.8 \cdot m$

Spring Problems

Hooke's Law: The force $f(x)$ required to stretch a spring x units beyond its natural length is $f(x) = kx$. Work is $\int_a^b f(x)\,dx$, where a and b are measured from the spring's natural length.

1. A force of 9 lbs is required to stretch a spring from its natural length of 6 in to a length of 8 in. Find the work done in stretching the spring
   (a) from its natural length to a length of 10 in (ans 3 ft-lb)
   (b) from a length of 7 in to a length of 9 in (ans 1.5 ft-lb)
2. A spring has natural length 1 m. A force of 24 N stretches it to a length of 1.8 m.
   (a) How much work is required to stretch the spring 2 meters beyond its natural length? (ans 60 J)
   (b) How far will a 45 N force stretch the spring? (ans 1.5 m)
3. It took 1800 J of work to stretch a spring from its natural length of 2 m to a length of 5 m. Find the spring's force constant. (ans 400 N/m)
4. p 479, number 5
5. p 479, number 6 (ans .31J)
6. p 479, number 7

Pumping a Liquid

1. A vertical cylindrical tank of diameter 3 ft and height 6 ft is full of water. Find the work required to pump all the water
   (a) over the top of the tank. (ans: 7952 ft-lbs)
   (b) through a pipe that rises to a height of 4 ft above the tank. (ans: 18,555 ft-lbs)
2. Same problem as above, except the tank is half full of water. (ans: a. 5964.12 ft-lb, b. 11,265.6 ft-lb)

3. An inverted conical tank with radius 5 ft and height 10 ft is filled within 2 ft of the top with olive oil. The olive oil weights 57 lb/ft^3. How much work does it take to pump the oil over the top of the tank? (30,561 ft-lb)

4. p 480, number 16 (ans: 339,292 ft-lb)

5. p 480, number 17a

6. An inverted conical tank has a radius of 10 ft and height 30 ft. It contains 12 ft of water. Find the work required to pump the water out over the top. (ans: 263,893.79 ft-lb)

7. How much work does it take to pump the water from a full upright circular cylindrical tank of radius 5 m and heigh 10 m to a lever 4 m above the top of the tank? (ans: 69,272,118 J)

8. The ends of an 8 ft long water trough are equilateral triangles having sides of length 2 ft. If the trough is full of water, find the work required to pump all of the water over the top. (ans: 500 ft-lb)

9. p 495, number 21a

Lifting an object and/or chain

1. A 5 lb hammer is lying at the bottom of an empty well that is 16 ft deep. A rope weighing $\frac{1}{16}$ lb per ft is used to lift the hammer to the surface. How much work is done? (ans: 88 ft-lb)

2. A 3 gal bucket full of water is at the bottom of a 100 ft well. How much work is required to lift this bucket to the top of the well, assuming the bucket weights 3 lbs when empty and the weight of the rope is insignificant? Use the fact that 1 gallon is approximately 0.134 ft^3. (ans: 2812.5 ft-lb)

3. A 3 gal bucket full of water is at the bottom of a 100 ft well. How much work is required to lift this bucket to the top of the well, assuming the bucket weights 3 lbs when empty and the rope weighs $\frac{1}{4}$ lb/ft? Use the fact that 1 gallon is approximately 0.134 ft^3. (ans: 4062.5 ft-lb)

4. A uniform cable 30 ft long and weighing a total of 60 pounds hangs vertically from a pulley system at the top of a building. A steel beam that weights 500 pounds is attached to the end of the cable. Find the work to pull it to the top. (ans: 15,900 ft-lb)

5. p 480, number 9

6. p 480, number 11

Hydrostatic Pressure and Force

1. A plate in the shape of a rectangle is submerged vertically in a tank of water. The width of the plate is 10 ft and the height is 8 ft. Find the force due to liquid pressure on one side of the plate if

   (a) the upper edge of the plate is touching the surface of the water. (ans 20,000 lbs)

   (b) if the plate is immersed at the bottom of a pool 14 ft deep. (ans: 49,920 lbs)

2. A flat right triangular plate with base 6 ft and height 3 ft is submerged vertically, base up, 2 ft below the surface of a swimming pool. Find the force exerted by the water against one side of the plate. (ans: 1687.5 lbs)

3. A trough having an isosceles trapezoidal cross section is full of water. If the trapezoid is 3 ft wide at the top, 2 ft wide at the bottom, and 2 ft deep, find the total force due to liquid pressure on one end of the trough. (ans: 291.67 lb)

4. A dam has a vertically submerged rectangular gate 5 ft wide and 4 ft high. The water level is 30 ft above the top of the gate. Find the total force on the gate. (ans: 40,000 lb)

5. A vertical gate in a dam has the shape of an isosceles trapezoid 8 ft across the top and 6 ft across the bottom, with height of 5 ft. What is the fluid force on the gate if the top of the gate is 4 ft below the surface of the water? (ans: 13,958.33 lb)

6. Find the fluid force on one end of a horizontal cylindrical tank of diameter 12 ft if it is half full of water. (ans: 9000 lb)

7. The ends of a trough are semicircles, each with radius of 2 ft. Find the force due to liquid pressure on one end of the trough, assuming the trough is full of water. (ans: 333.33 lb)

8. p 481, number 23

9. p 481, number 25

Centroids

Sketch the region bounded by the curves. Find the mass (m), M_x, M_y and the centroid $(\bar{x}, \bar{y})$

1. $y = x^3$, $y = 0$, $x = 1$ (ans: $\frac{1}{4}\rho$, $\frac{1}{5}\rho$, $\frac{1}{14}\rho$, $(\frac{4}{5}, \frac{2}{7})$)
2. $y = 4 - x^2$, $y = 0$ (ans: $\frac{32}{3}\rho$, $\frac{256}{15}\rho$, 0, $(0, \frac{8}{5})$)
3. $y^2 = x$, $2y = x$ (ans: $\frac{4}{3}\rho$, $\frac{4}{3}\rho$, $\frac{32}{15}\rho$, $(\frac{8}{5}, 1)$)
4. $y = 1 - x^2$, $x - y = 1$ (ans: $\frac{9}{2}\rho$, $-\frac{27}{10}\rho$, $-\frac{9}{4}\rho$, $(-\frac{1}{2}, -\frac{3}{5})$)
5. $x = y^2$, $x - y = 2$ (ans: $\frac{9}{2}\rho$, $\frac{9}{4}\rho$, $\frac{36}{5}\rho$, $(\frac{8}{5}, \frac{1}{2})$)
6. $y = 6 - x^2$, $y = 3 - 2x$ (ans: $\frac{32}{3}\rho$, $\frac{832}{15}\rho$, $\frac{32}{3}\rho$, $(1, 2.6)$)