Partial Fracs

22. $$\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$$

P.F

$$\frac{x-4}{\underset{(x-2)(x-3)}{x^2-5x+6}} = \frac{A}{x-2} + \frac{B}{x-3}$$

So $x-4 = A(x-3) + B(x-2)$

Root Method

	LHS		RHS
$x=3$	-1	$=$	B
$x=2$	-2	$=$	$-A$

$\Rightarrow B=-1$ & $A=2$

$$\int_0^1 \frac{x-4}{x^2-5x+6}\,dx = \int_0^1 \frac{2}{x-2}\,dx - \int_0^1 \frac{1}{x-3}\,dx$$

$$= 2\ln|x-2|\Big|_0^1 - \ln|x-3|\Big|_0^1$$

$$= [2\ln(1) - 2\ln(2)] - [\ln(2) - \ln(3)]$$

$$= \underline{-3\ln(2) + \ln(3)}$$

24. $$\int \frac{x^2+2x-1}{x^3-x}\,dx$$

P.F. $$\frac{x^2+2x-1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x-1)} \Rightarrow A(x+1)(x-1) + B(x)(x-1) + C(x)(x+1)$$

Root Method

$x^2+2x-1 = A(x+1)(x-1) + Bx(x-1) + Cx(x+1)$

	LHS		RHS
$x=0$	-1	$=$	$-A$
$x=1$	2	$=$	$2C$
$x=-1$	-2	$=$	$2B$

$\Rightarrow A=1$, $B=-1$, $C=1$

$$\int \frac{x^2+2x-1}{x^3-x}\,dx = \int \frac{1}{x}\,dx - \int \frac{1}{x+1}\,dx + \int \frac{1}{x-1}\,dx$$

$$= \underline{\ln|x| - \ln|x+1| + \ln|x-1| + C}$$

P.F. con't

12. $\displaystyle\int_0^1 \frac{x-1}{x^2+3x+2}\,dx$

$(x+2)(x+1)$

P.F.

$$\frac{x-1}{x^2+3x+2} = \frac{A}{(x+2)} + \frac{B}{(x+1)} \Rightarrow x-1 = A(x+1) + B(x+2)$$

Root Method	LHS		RHS	
$x=-2$	-3	$=$	$-A$	$\Rightarrow A=3$
$x=-1$	-2	$=$	B	$B=-2$

$$\int_0^1 \frac{x-1}{x^2+3x+2}\,dx = \int_0^1 \frac{3}{x+2}\,dx - \int_0^1 \frac{2}{x+1}\,dx$$

$$= 3\ln|x+2|\Big|_0^1 - 2\ln|x+1|\Big|_0^1$$

$$= [3\ln(3) - 3\ln(2)] - [2\ln(2) - 2\ln(1)]$$

$$= \underline{3\ln(3) - 5\ln(2)}$$

20. $\displaystyle\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx$

P.F.

$$\frac{x^2-5x+16}{(2x+1)(x-2)^2} = \frac{A}{2x+1} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$x^2-5x+16 = A(x-2)^2 + B(2x+1)(x-2) + C(2x+1)$$

Root Method	LHS		RHS	
$x=2$	10	$=$	$5C$	$\Rightarrow C=2$
$x=-1/2$	$75/4$	$=$	$25/4\,A$	$A=3$

Must compare coeff. Consider x^2-term

LHS : x^2

RHS : $Ax^2 + 2Bx^2 = (A+2B)x^2$ $\Big\}$ so $1 = A+2B$

$\Downarrow$

$1 = 3+2B$

$-2 = 2B$

$\Downarrow$

$B=-1$

$$\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx = \int \frac{3}{2x+1}\,dx - \int \frac{1}{(x-2)}\,dx + \int \frac{2}{(x-2)^2}\,dx$$

$$= \underline{\frac{3}{2}\ln|2x+1| - \ln|x-2| - \frac{2}{(x-2)} + C}$$

26. $\int \frac{x^2+x+1}{(x^2+1)^2}\,dx$

P.F

$$\frac{x^2+x+1}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2}$$

$$x^2+x+1 = (Ax+B)(x^2+1) + (Cx+D)$$

Must use comparing coeff. method

$$x^2+x+1 = Ax^3+Bx^2+(A+C)x+(B+D)$$

Since no LHS x^3 term, $\boxed{A=0}$

$\Downarrow$

$(A+C)=1$ so $\boxed{C=1}$

LHS x^2-term has coeff. 1 $\Rightarrow \boxed{B=1}$

$\Downarrow$

$(B+D)=1$

$\boxed{D=0}$

$$\int \frac{x^2+x+1}{(x^2+1)^2}\,dx = \int \frac{1}{x^2+1}\,dx + \int \frac{x}{(x^2+1)^2}\,dx$$

$$= \arctan(x) + \underbrace{\int \frac{x}{(x^2+1)^2}\,dx}_{=} \rightarrow \quad u = x^2+1$$

$du = 2x\,dx \Rightarrow dx = \frac{du}{2x}$

$$\frac{1}{2}\int \frac{1}{u^2}\,du = -\tfrac{1}{2}u^{-1}+C = -\frac{1}{2}\cdot\frac{1}{x^2+1}+C$$

$$= \underline{\arctan(x) - \tfrac{1}{2}\cdot\frac{1}{x^2+1} + C}$$