Name Row

Test 4B C. Buell

Show all work. You must show work to get credit. No calculators. No cellphones. You will have 50 minutes. Good luck.

1. (20 points) Use a series test to determine convergence or divergence for the following.

   (a) $\sum_{n=1}^{\infty} \frac{n}{n^3+4}$

   (b) $\sum_{n=1}^{\infty} \frac{(-5)^{2n}}{n^2 9^n}$

2. (12 points) What is the interval of convergence of the following series?

   $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n3^n}$

3. (10 points) Use the definition of a Taylor Series to find the FIRST FOUR terms ($T_4(x)$ meaning n=0,1,2,3) of the series for $f(x) = x^{2/3}$ centered at 1. Do not write in summation notation, just write out the first four terms of the series.

4. (6 points) $f(x)$ has the following Taylor series expansion.

   $f(x) = (x-3) + (x-3)^2 + \frac{1}{3}(x-3)^3 + \frac{2}{15}(x-3)^4 + \frac{7}{31}(x-3)^5 + ...$

   What is $f^{(4)}(3)$? Find it without finding $f^{(n)}(x)$. Show your work.

5. (36 points) Use your knowledge of the Maclaurin series of $e^x, sin(x), cos(x), \frac{1}{1-x}$ to obtain the Maclaurin series for the following. Write your answer in summation form and completely simplified.

   (a) $f(x) = xe^{x^2/3}$

   (b) $f(x) = \frac{4x}{(3-x)^2}$

   (c) Evaluate the indefinite integral as an infinite series. $\int \frac{sin(3x)}{x} dx$

6. (16 points) Find the sum of the following series (be careful of details)

   (a) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{5^{2n}(2n)!}$

   (b) $\sum_{n=1}^{\infty} \frac{(ln(3))^n}{n!}$

Test 4B Ans.

1. a) $\lim_{n\to\infty} \dfrac{\frac{1}{n^2}}{\frac{n}{n^3+4}} = \lim_{n\to\infty} \dfrac{n^3+4}{n^3} = 1$

$0 < 1 < \infty$ & $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges $p = 2 > 1$

$\therefore \sum_{n=1}^{\infty} \frac{n}{n^3+4}$ converges

b) $\lim_{n\to\infty} \left| \dfrac{\frac{(-5)^{2(n+1)}}{(n+1)^2 9^{n+1}}}{\frac{(-5)^{2n}}{n^2 9^n}} \right| = \lim_{n\to\infty} \left| \dfrac{(-5)^{2n+2} n^2 9^n}{(-5)^{2n}(n+1)^2 9^{n+1}} \right|$

$= \lim_{n\to\infty} \left| \dfrac{(-5)^2 n^2}{(n+1)^2 9} \right| = \dfrac{25}{9} > 1$ $\therefore$ diverges by Ratio Test.

2. $\lim_{n\to\infty} \left| \dfrac{\frac{(x+2)^{n+1}}{(n+1)3^{n+1}}}{\frac{(x+2)^n}{n(3^n)}} \right| = \lim_{n\to\infty} \left| \dfrac{(x+2)^{n+1} n \, 3^n}{(x+2)^n (n+1) 3^{n+1}} \right|$

$= \dfrac{|x+2|}{3} < 1 \Rightarrow |x+2| < 3$

$-3 < x+2 < 3$

$-5 < x < 1$

$\underline{x=1}$ $\sum_{n=1}^{\infty} \frac{3^n}{n3^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges $p=1$, Harmonic

$\underline{x=-5}$ $\sum_{n=1}^{\infty} \frac{(-3)^n}{n3^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ A.S.T. 1) $\lim_{n\to\infty} \frac{1}{n} = 0$ 2) $\frac{1}{n+1} < \frac{1}{n}$ $\therefore$ converges

$[-5, 1)$

3.

	at 1	c_n
$f(x) = x^{2/3}$	1	1
$f'(x) = \frac{2}{3}x^{-1/3}$	$\frac{2}{3}$	$\frac{2}{3}$
$f''(x) = -\frac{2}{9}x^{-4/3}$	$-\frac{1}{9}$	$\frac{-2}{9\cdot 2!}$
$f'''(x) = \frac{8}{27}x^{-7/3}$	$\frac{8}{27}$	$\frac{8}{27\cdot 3!}$

$$f(x) \approx T_4(x) = 1 + \frac{2}{3}(x-1) - \frac{2}{9\cdot 2!}(x-1)^2 + \frac{8}{27\cdot 3!}(x-1)^3$$

4. $$\frac{2}{15} = \frac{f^{(4)}(3)}{4!} \Rightarrow f^{(4)}(3) = \boxed{\frac{2\cdot 4!}{15}} = \boxed{\frac{48}{15}}$$

5. a) $f(x) = xe^{x^2/3}$

$$e^x \xrightarrow[\text{by } \frac{x^2}{3}]{\text{replace } x} e^{x^2/3} \xrightarrow[x]{\text{mult by}} xe^{x^2/3}$$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} \longrightarrow \sum_{n=0}^{\infty} \frac{\left(\frac{x^2}{3}\right)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{3^n n!} \longrightarrow \underline{\sum_{n=0}^{\infty} \frac{x^{2n+1}}{3^n n!}}$$

b) $f(x) = \frac{4x}{(3-x)^2}$

$$\frac{1}{1-x} \xrightarrow[\frac{1}{3}]{\text{mult by}} \frac{1}{3-3x} \xrightarrow[\text{in for } x]{\text{sub } \frac{x}{3}} \frac{1}{3-x} \xrightarrow{\text{diff}} \frac{1}{(3-x)^2} \xrightarrow[\text{by } 4x]{\text{mult}} \frac{4x}{(3-x)^2}$$

$$\sum_{n=0}^{\infty} x^n \longrightarrow \frac{1}{3}\sum_{n=0}^{\infty} x^n \longrightarrow \frac{1}{3}\sum_{n=0}^{\infty} \frac{x^n}{3^n} \longrightarrow \frac{1}{3}\sum_{n=1}^{\infty} \frac{nx^{n-1}}{3^n} \longrightarrow \underline{\sum_{n=1}^{\infty} \frac{4nx^n}{3^{n+1}}}$$

c) $$\sin(3x) = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+1}}{(2n+1)!}$$

$$\frac{\sin(3x)}{x} = \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n}}{(2n+1)!}.$$

$$\int \frac{\sin(3x)}{x}\,dx = \underline{K + \sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+1}}{(2n+1)(2n+1)!}}$$

6. a) $$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^n}{5^{2n}(2n)!} = \cos\left(\frac{\sqrt{\pi}}{5}\right)$$

b) $$\sum_{n=1}^{\infty} \frac{(\ln(3))^n}{n!} = \sum_{n=0}^{\infty} \frac{\ln(3)^n}{n!} - 1$$

$$= e^{\ln(3)} - 1 = \underline{2}$$