Name Row

MA 241-004 Test 3A C. Buell

Show all work. You must show work to get credit. No calculators. No cellphones. You will have 50 minutes. Good luck.

1. (3 points) $4! =$

2. (10 points) Solve the following initial value problem.

$$y'' - 6y' + 10y = 0 \text{ where } y(0) = 4 \text{ and } y'(0) = 2$$

3. (8 points) What is the correct guess for the particular solution of the following differential equation? DO NOT SOLVE FOR THE COEFFICIENTS.

$$y'' - 6y' = 1 + xe^{6x} + cos(x)$$

4. (14 points) Use method of undetermined coeffients to find the general solution of the following.

$$y'' - 2y' + y = 2sin(t)$$

5. (14 points) Use variation of parameters to solve the following. You must use variation of parameters to receive credit.

$$y'' - y = 4e^{-x}$$

6. (11 points) A spring with a natural length of 3 meters hangs vertically. Typically it takes 16N of force to stretch the spring to 5m. A mass of 2kg attached to the bottom and the spring is critically damped ($c > 0$). The spring is compressed to 1m and released with a downward velocity of 1m/sec. Write the initial conditions and write the differential equation that describes the motion of the spring in terms of the function $x(t)$ (ie. Write the differential equation that you would solve in order to determine the position function $x(t)$, but don't solve for $x(t)$). DO NOT SOLVE FOR $x(t)$, JUST WRITE THE DIFFERENTIAL EQUATION YOU WOULD SOLVE AND WRITE OUT INITIAL CONDITIONS.

7. ~~([illegible] points) Let y_p be the particular solution of the non-homogeneous differential equation $ay'' + by' + cy = G(x)$ and y_c be the general solution to the associated homogeneous differential equation. Prove that $(y_c + y_p)$ solves the non-homogeneous differential equation.~~

8. (12 points) Determine whether the sequence converges or diverges. If it converges, find its limit. If it diverges, justify your answer.

   (a) $a_n = 1$

   (b) $\left\{\frac{2^n}{5^{n+1}}\right\}_{n=0}^{\infty}$

9. (21 points) Determine whether the series converges or diverges. If it converges justify your answer and find its sum. If it diverges, justify your answer.

   (a) $\sum_{n=1}^{\infty}(.5)^{n-1}$

   (b) $\sum_{n=1}^{\infty}\frac{2n^2+1}{3n-3}$

   (c) $2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27}\cdots$

3A Answers

1) $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

2) $y'' - 6y' + 10y = 0 \qquad y(0) = 4 \quad \& \quad y'(0) = 2$

$$r^2 - 6r + 10 = 0$$

$$r = \frac{6 \pm \sqrt{36-10}}{2} = 3 \pm i$$

$$y_c = e^{3x}(c_1\cos(x) + c_2\sin(x))$$

$$y_c' = 3e^{3x}(c_1\cos(x) + c_2\sin(x)) + e^{3x}(-c_1\sin(x) + c_2\cos(x))$$

$$4 = y(0) = c_1$$

$$2 = y'(0) = 3c_1 + c_2 = 12 + c_2 \qquad c_2 = -10$$

$$y_c = e^{3x}(4\cos(x) - 10\sin(x))$$

3) $y'' - 6y' = 1 + xe^{6x} + \cos(x)$

$r^2 - 6r = 0$

$r = 0, 6$

$$y_c = c_1 + c_2 e^{6x}$$

$$y_p = Ax + (Bx + C)xe^{6x} + D\cos(x) + E\sin(x)$$

5) $y'' - y = 4e^{-x}$

$$y_c = c_1 e^x + c_2 e^{-x}$$

$r^2 - 1 = 0$

$r = 1, -1$

$$u_1' e^x + u_2' e^{-x} = 0$$

$$+ \quad u_1' e^x - u_2' e^{-x} = 4e^{-x}$$

$$2u_1' e^x = 4e^{-x}$$

$$u_1' = 2e^{-2x} \Rightarrow u_1 = -e^{-2x}$$

$$2e^{-2x}e^{x} + u_2' e^{-x} = 0$$

$$u_2' = -2 \quad \Rightarrow u_2 = -2x$$

$$y_g = c_1 e^{x} + c_2 e^{-x} - e^{-2x}(e^{x}) - 2xe^{-x}$$

$$= \quad '' \quad '' \quad - 2xe^{-x}$$

4. $y'' - 2y' + y = 2\sin(t)$

$$r^2 - 2r + 1 = 0$$

$$r = -1, -1$$

$$y_c = c_1 e^{-t} + c_2 te^{-t}$$

$$y_p = A\cos(t) + B\sin(t)$$

$$y_p' = -A\sin(t) + B\cos(t)$$

$$y_p'' = -A\cos(t) - B\sin(t)$$

$$-A\cos(t) - B\sin(t) + 2A\sin(t) - 2B\cos(t) + A\cos(t) + B\sin(t) = 2\sin(t)$$

$$2A\sin(t) - 2B\cos(t)$$

$$B = 0 \qquad A = 1$$

$$y_g = c_1 e^{-t} + c_2 te^{-t} + \cos(t)$$

6. $16 = k \cdot 2 \Rightarrow k = 8$

$$c^2 - 4(2)(8) = 0 \qquad c = 8$$

$$2x'' + 8x' + 8x = 0 \qquad \begin{aligned} x(0) &= -2 \\ x'(0) &= 1 \end{aligned}$$

8. (a) $a_n = 1$ converges to 1

(b) $\frac{1}{5}\left\{\left(\frac{2}{5}\right)^n\right\}_{n=0}^{\infty}$ converges $\left(\frac{2}{5}\right) < 1$ to 0

9. (a) converges $.5 < 1$

$$\rightarrow \frac{1}{1-.5} = 2$$

(b) $\lim\limits_{n\to\infty} \frac{2n^2+1}{3n-3} = \infty$ so by divergence test.

$\sum\limits_{n=1}^{\infty} \frac{2n^2+1}{3n-3}$ diverges

(c) $2 - \frac{4}{3} + \frac{8}{9} - \frac{16}{27}$

$$= \sum_{n=1}^{\infty} \frac{2^n}{(-3)^{n-1}} = \sum_{n=1}^{\infty} 2\left(\frac{2}{-3}\right)^{n-1} \quad \left|-\frac{2}{3}\right| < 1$$

converges to

$$\frac{2}{1+\frac{2}{3}} = \frac{2}{5/3} = 6/5$$