

$$1) f(x) = \frac{x}{1-x}$$

$$\frac{1}{1-x} \text{ multiply by } x \Rightarrow \sum_{n=0}^{\infty} x^n \rightarrow \boxed{\sum_{n=0}^{\infty} x^{n+1}} \quad |x| < 1$$

$$2) f(x) = \frac{1}{1+64x^2}$$

$$\frac{1}{1-x} \xrightarrow[\text{with } -64x^2]{\text{replace } x} \frac{1}{1+64x^2}$$

$$\sum_{n=0}^{\infty} x^n \Rightarrow$$

$$\sum_{n=0}^{\infty} (-64x^2)^n = \boxed{\sum_{n=0}^{\infty} (-64)^n x^{2n}}$$

$$| -64x^2 | < 1 \Rightarrow |x| < \frac{1}{8}$$

$$3) f(x) = \frac{x}{1+8x}$$

$$\frac{1}{1-x} \xrightarrow[\text{with } -8x]{\text{replace } x} \text{ multiply by } x \rightarrow \frac{x}{1+8x}$$

$$\sum_{n=0}^{\infty} x^n \rightarrow$$

$$\sum_{n=0}^{\infty} (-8x)^n = \sum_{n=0}^{\infty} (-8)^n x^n \rightarrow \boxed{\sum_{n=0}^{\infty} (-8)^n x^{n+1}}$$

$$| -8x | < 1 \Rightarrow |x| < \frac{1}{8}$$

$$4) f(x) = \frac{x^2}{(1-2x)^2}$$

$$\frac{1}{1-x} \xrightarrow{\text{diff}} \frac{1}{(1-x)^2} \xrightarrow[\text{w/ } 2x]{\text{replace } x} \frac{1}{(1-2x)^2} \xrightarrow[\text{by } x^2]{\text{mult.}} \frac{x^2}{(1-2x)^2}$$

$$\sum_{n=0}^{\infty} x^n \rightarrow$$

$$\sum_{n=1}^{\infty} n x^{n-1} \rightarrow$$

$$\sum_{n=1}^{\infty} n(2x)^{n-1} \rightarrow$$

$$\sum_{n=1}^{\infty} x^2 n 2^{n-1} x^{n-1}$$

$$= \boxed{\sum_{n=1}^{\infty} n 2^{n-1} x^{n+1}}$$

$$|2x| < 1 \Rightarrow |x| < \frac{1}{2}$$

$$5) f(x) = \arctan\left(\frac{x}{6}\right)$$

$$\frac{1}{1-x} \xrightarrow{\text{reg. by } 1} \frac{1}{-x^2} \frac{1}{1+x^2} \xrightarrow{\text{int.}} \arctan(x) \xrightarrow{\text{subs } x, x/6} \arctan\left(\frac{x}{6}\right)$$

$$\sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n} \rightarrow \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \rightarrow \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{x}{6}\right)^{2n+1}}{2n+1}$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{6^{2n+1} (2n+1)}}$$

$$1 - \left(\frac{x}{6}\right)^2 < 1 \Rightarrow |x| < 6$$

$$6) \int \frac{x}{1+x^5} dx$$

$$\sum_{n=0}^{\infty} x \cdot (-x^5)^n = \sum_{n=0}^{\infty} (-1)^n x^{5n+1}$$

$$\text{Integrate: } \boxed{\left( \sum_{n=0}^{\infty} (-1)^n \frac{x^{5n+2}}{5n+2} \right) + K}$$