

MATHEMATICS DEPARTMENT
North Carolina State University

ALGEBRA AND COMBINATORICS SEMINAR

Friday, September 26, 2008

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A generalization of the Euler–Glaisher partition bijection

ABSTRACT: In 1748, Euler published his *Introductio in Analysin Infinitorum*. Chapter 16 of this work is the first systematic study of integer partitions in the mathematical literature. In it, he introduces infinite product generating functions and uses them to derive what is now known as Euler’s partition identity, an English translation of which reads as follows: “The number of different ways a given number can be expressed as the sum of different whole numbers is the same as the number of ways in which the same number can be expressed as the sum of odd numbers, whether the same or different.” In modern terminology, the preceding is rephrased as “the number of partitions of n into distinct parts equals the number of partitions of n into odd parts.”

In 1883, J.W.L. Glaisher published the first bijective proof of Euler’s partition identity, along with a natural generalization: “the number of partitions of n where no part appears more than $m - 1$ times equals the number of partitions of n where no part is divisible by m .”

By combining a construction of P.A. MacMahon called “partitions of infinity” and knowledge of G.E. Andrews’ “partition ideals of order 1” with Glaisher’s bijective proof of Euler’s identity, we are led to discover a large class of partition identities with straightforward bijective proofs.

This is joint work with James Sellers and Gary Mullen of Penn State.

All terms will be defined and illustrated with concrete examples, so the required mathematical background will be minimal.

4:00 - 4:50 pm HA 335

Faculty, students and post-docs are encouraged to attend.