Vertex Operators and Quantum Cohomology of the Grassmannians

ABSTRACT: The (small) quantum cohomology was introduced by Ruan and Tian and later developed by Kontsevich and Manin. Algebraically it is a deformation of the usual cohomology. The ring structure of quantum cohomology of homogeneous spaces are given by the three-point genus zero Gromov–Witten invariants, which count the number of rational curves meeting general Schubert varieties. It is difficult to compute those invariants in general and only a few examples are known completely which often involve the more sophisticated intersection theory on moduli spaces. In this work we provide a new method to compute the quantum cohomology of the Grassmannians using vertex algebras. This is joint work with I. Frenkel.

We formulate the ring of quantum cohomology of the Grassmannians in terms of certain Fock space of the affine Lie algebra of \( sl(2) \). Each class of the Schubert variety will be realized by certain vertex operator, and we show that the algebraic structure of the quantum cohomology is also governed by special vertex operators. One direct consequence of our approach is a new effective way to compute the Gromov–Witten invariants.

3:00 - 3:50 pm    HA 335

Faculty and Students are invited to attend.