

MATHEMATICS DEPARTMENT
North Carolina State University

ALGEBRA SEMINAR

Friday, March 31, 2006

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UNC Chapel Hill

On Cachazo–Douglas–Seiberg–Witten Conjecture
for Simple Lie Algebras

ABSTRACT: Let \mathfrak{g} be a finite dimensional simple Lie algebra over the complex numbers \mathbb{C} . Consider the exterior algebra $R := \bigwedge(\mathfrak{g} \oplus \mathfrak{g})$ on two copies of \mathfrak{g} . Then, the algebra R is bigraded under

$$R^{p,q} = \bigwedge^p(\mathfrak{g}) \otimes \bigwedge^q(\mathfrak{g}).$$

The diagonal adjoint action of \mathfrak{g} gives rise to a \mathfrak{g} -algebra structure on R compatible with the bigrading. There are three ‘standard’ copies of the adjoint representation \mathfrak{g} in R^2 . Let J be the (bigraded) ideal of R generated by these three copies of \mathfrak{g} (in R^2) and define the bigraded \mathfrak{g} -algebra

$$A := R/J.$$

The Killing form gives rise to a \mathfrak{g} -invariant $S \in A^{1,1}$.

Motivated by supersymmetric gauge theory, Cachazo–Douglas–Seiberg–Witten made the following conjecture.

- Conjecture. (i) The subalgebra $A^{\mathfrak{g}}$ of \mathfrak{g} -invariants in A is generated, as an algebra, by the element S .
(ii) $S^h = 0$.
(iii) $S^{h-1} \neq 0$, where h is the dual Coxeter number of \mathfrak{g} .

The aim of this talk is to give a uniform proof of the above conjecture part (i). This theorem is proved by using Garland’s result on the Lie algebra cohomology of $\hat{\mathfrak{u}} := \mathfrak{g} \otimes t\mathbb{C}[t]$; Kostant’s result on the ‘diagonal’ cohomology of $\hat{\mathfrak{u}}$ and its connection with abelian ideals in \mathfrak{b} ; and a certain deformation of the singular cohomology of \mathcal{Y} introduced by Belkale–Kumar.

3:00 - 3:50 pm HA 335

Faculty and Students are invited to attend.