

## NCSU Mathematics Department

### MA 792B-001 – Vertex Algebras

Spring 2008, TH 10:15-11:30 am, HA 377

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**Course Webpage:** <http://www4.ncsu.edu/~bnbakalo/ma792b.html>

#### Tentative Plan of Lectures:

1. Vertex algebras and the Monster (a short historical talk). The definition of vertex algebra.
2. Holomorphic vertex algebras. Formal distributions. Locality of a formal distribution in two variables. Formal delta function.
3. Local formal distributions with values in a Lie algebra. Examples: Heisenberg, affine Kac-Moody, and Virasoro algebras.
4. Properties of local formal distributions.  $n$ -th brackets and  $\lambda$ -bracket. The definition of vertex Lie algebra.
5. Formal Fourier transform. Examples of vertex Lie algebras. Lie superalgebras. Free fermions.
6. Formal distributions with values in an associative algebra. Formal expansions. Normally ordered product.
7. Locality and polylocality. Normally ordered product and  $n$ -th products of fields. Dong's Lemma.
8. Linear field algebras. Any local linear field algebra is a vertex algebra.
9. The definition of vertex algebra. Consequences of the vacuum and translation axioms. Skew-symmetry.
10. Uniqueness Theorem.  $n$ -th product identity and commutator formula. Derivations of vertex algebras.
11. Jacobi identity. Equivalence of two definitions of vertex algebra.

12. Consequences and special cases of the Jacobi identity. Borcherds identity.
13. Generating a vertex algebra from a collection of local fields: Existence Theorem.
14. Representations of the Heisenberg Lie algebra. Free boson vertex algebra.
15. Computations in the free boson vertex algebra. Virasoro algebra. Graded and conformal vertex algebras.
16. Neutral free fermion vertex algebra.
17. Charged free fermions. First part of the boson-fermion correspondence.
18. Vertex algebras associated to a 1-dimensional lattice. Second part of the boson-fermion correspondence.
19. Lattice vertex algebras.
20. Lattice vertex algebras and affine vertex algebras at level one.
21. Vertex algebras related to a Lie algebra of local formal distributions. Affine vertex algebras.
22. Affine vertex algebras: Sugawara construction.
23. Modules over vertex algebras: definition in terms of  $n$ -th product identity.
24. Modules over vertex algebras: associativity and Jacobi identity.
25. Consequences of the definition of a module: translation invariance, locality, generating a module from a subset. Uniqueness theorem for modules.
26. Graded modules over graded vertex algebras. Action on the lowest weight space and Zhu's  $*$ -product.
27. Zhu algebra of a graded vertex algebra.
28. Correspondence between vertex algebra modules and modules over its Zhu algebra.