1. Let $V$ be the complex vector space of all $2 \times 2$ matrices with trace zero. Consider the ordered basis $B = \{E, F, H\}$ of $V$, where
\[
E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.
\]
Define the map $T: V \to V$ by $T(A) = EA - AE$ for $A \in V$.

a. Verify that $T$ is a linear operator.

b. Find the matrix of $T$ relative to the ordered basis $B$ of $V$.

c. Determine the range, null space, rank and nullity of $T$.

d. Is $T$ nonsingular? Is $T$ invertible?

2. Consider the real vector space $V$ of polynomials from $\mathbb{R}[x]$ of degree $\leq 2$. Let $f_0$, $f_1$, $f_2$ be the linear functionals on $V$ defined by: $f_n(p) = p^{(n)}(n)$ for $p \in V$, $n = 0, 1, 2$.

a. Find a basis $\{p_0, p_1, p_2\}$ for $V$ dual to the basis $\{f_0, f_1, f_2\}$ for $V^*$.

b. Let $D = d/dx$ be the differentiation operator on $V$. Find the coordinates of $D^4f_0$ relative to the ordered basis $\{f_0, f_1, f_2\}$.

3. Let $W_1$ and $W_2$ be subspaces of a finite-dimensional vector space $V$. Prove that if $W_1 \subseteq W_2$, then $W_1^0 \supseteq W_2^0$.

4. Given the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$, evaluate $\det(A)$, $\det(2A)$, $\det(A^{-1})$, $\det(AA^t)$.