For each problem, show all work and all steps of your solution. If your solution is incomplete or contains errors, you will not receive full credit, even if your final answer is correct. Partial credit will be given for any piece which is a part of a correct solution.

1. (20 points) For two vectors $\mathbf{x} = (x_1, x_2, x_3)^T$, $\mathbf{y} = (y_1, y_2, y_3)^T \in \mathbb{R}^3$, compute the following matrix products, whenever they exist:
   (a) $\mathbf{xy}$;  
   (b) $\mathbf{x}^T \mathbf{y}$;  
   (c) $\mathbf{xy}^T$.

2. (50 points) Consider the matrices $A = \begin{pmatrix} 2 & 2 & 1 \\ 10 & 7 & 4 \\ -4 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & 1 \\ 6 & 3 & 2 \\ -4 & -1 & 1 \end{pmatrix}$.
   (a) Find the LU factorization of $A$.
   (b) Compute the determinant of $A$.
   (c) Solve the homogeneous system of linear equations $A\mathbf{x} = 0$.
   (d) Find an elementary matrix $E$ such that $B = EA$.
   (e) Is the matrix $B$ invertible? Why?

3. (30 points) Consider the vectors $\mathbf{v}_1 = (1, 1, 2, 5)^T$, $\mathbf{v}_2 = (1, 1, 1, 4)^T$ and $\mathbf{v}_3 = (1, -1, 1, 2)^T \in \mathbb{R}^4$.
   (a) Is $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ a spanning set for $\mathbb{R}^4$?
   (b) Are the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ linearly independent?