Show all work.

1. (20 points)
   A sample of radioactive material disintegrates from 6 grams to 2 grams in 50 days.
   (a) Find a formula for \( y(t) \), the amount of the radioactive material left after \( t \) days.
   (b) After how many days will just 1 gram remain?

2. (25 points)
   Sketch the graphs of the constant solutions and the solution curves corresponding to the given initial conditions:
   (a) \( y' = 8 - 2y;\ y(0) = 2,\ y(0) = 4.\)
   (b) \( y' = -(y + 3)(y - 5);\ y(0) = -4,\ y(0) = 2,\ y(0) = 6.\)

3. (30 points)
   (a) Solve the differential equation \( y' = te^{2y}.\)
   (b) Solve the given differential equation with initial condition \( y' = y^2 - e^{3t}y^2,\ y(0) = 1.\)

4. (25 points)
   A pond presently has 5,000 fish. The birth rate of the fish is 3\% month and the death rate is 1\% per month.
   (a) Write a differential equation for \( y = f(t) \), the number of fish in the pond at time \( t \).
   (b) The farmer who owns the pond wants to harvest \( M \) fish per month. Write down a new differential equation satisfied by \( y = g(t) \), the number of fish in the pond after \( t \) months. What is the maximum number of fish he can harvest each month and ensure that the fish population will not die out?