1. a) The series is geometric with a first term \(a = \frac{2}{5}\) and a ratio \(r = \frac{-2/25}{2/5} = -1/5\). Since \(|r| = 1/5 < 1\), the series is convergent. Its sum is
\[
\frac{a}{1 - r} = \frac{2/5}{1 - (-1/5)} = \frac{2/5}{6/5} = \frac{1}{3}.
\]

b) On the first day, shortly after the first dose of medicine, the patient will have \(D\) mgs of drug in the body. On the second day, of these \(D\) mgs only 20\% will be left, which makes it \(D(.2)\), and then another \(D\) mgs will be added. So, on the second day, there are \(D + D(.2)\) mgs of drug in the body. Similarly, on the third day, there are \(D + (D + D(.2))(.2) = D + D(.2) + D(.2)^2\) mgs of drug in the body. In the long run, the amount of drug in the body approaches the infinite series \(D + D(.2) + D(.2)^2 + D(.2)^3 + \cdots\). This is a geometric series with a first term \(a = D\) and a ratio \(r = .2\). Its sum is
\[
\frac{a}{1 - r} = \frac{D}{1 - .2} = \frac{D}{.8}. \quad \text{We want this amount to be 4 mgs; so we get } \frac{D}{.8} = 4 \quad \text{and } D = 4 \times .8 = 3.2.
\]

2. a) The formula for the third Taylor polynomial of \(f(x)\) at \(x = a\) is:
\[
p_3(x) = f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \frac{f'''(a)}{3!}(x - a)^3,
\]
where \(1! = 1, 2! = 1 \times 2 = 2, 3! = 1 \times 2 \times 3 = 6\).

In our case, \(f(x) = \sqrt{x}\) and \(a = 4\). We compute:
\[
f(x) = \sqrt{x} = x^{1/2},
\]
\[
f'(x) = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2} \quad \text{(using the power rule)},
\]
\[
f''(x) = \frac{1}{2}(-\frac{1}{2})x^{-1/2-1} = -\frac{1}{4}x^{-3/2},
\]
\[
f'''(x) = -\frac{1}{4}(-\frac{1}{2})x^{-3/2-1} = \frac{3}{8}x^{-5/2}.
\]
\[ f(4) = 4^{\frac{1}{2}} = \sqrt{4} = 2, \]
\[ f'(4) = \frac{1}{2} 4^{-\frac{1}{2}} = \frac{1}{2} (4^{\frac{1}{2}})^{-1} = \frac{1}{2} 2^{-1} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}, \]
\[ f''(4) = -\frac{1}{4} 4^{-\frac{3}{2}} = -\frac{1}{4} (4^{\frac{1}{2}})^{-3} = -\frac{1}{4} 2^{-3} = -\frac{1}{4} \cdot \frac{1}{8} = -\frac{1}{32}, \]
\[ f'''(4) = \frac{3}{8} 4^{-\frac{5}{2}} = \frac{3}{8} (4^{\frac{1}{2}})^{-5} = \frac{3}{8} 2^{-5} = \frac{3}{8} \cdot \frac{1}{32} = \frac{3}{256}. \]

We find that the third Taylor polynomial of \( \sqrt{x} \) at \( x = 4 \) is:

\[ p_3(x) = 2 + \frac{1/4}{1} (x - 4) + \frac{-1/32}{2} (x - 4)^2 + \frac{3/256}{6} (x - 4)^3 \]
\[ = 2 + \frac{1}{4} (x - 4) - \frac{1}{64} (x - 4)^2 + \frac{1}{512} (x - 4)^3. \]

b) To approximate \( \sqrt{3.5} \), we use \( p_3(3.5) \). Since \( 3.5 - 4 = -0.5 = -\frac{1}{2} \), we get

\[ p_3(3.5) = 2 + \frac{1}{4} \left( \frac{-1}{2} \right) - \frac{1}{64} \left( \frac{-1}{2} \right)^2 + \frac{1}{512} \left( \frac{-1}{2} \right)^3 = 2 - \frac{1}{8} - \frac{1}{256} - \frac{1}{4096} \]
\[ = \frac{2 \times 4096 - 512 - 16 - 1}{4096} = \frac{7663}{4096} = 1.8708\ldots. \]

3. a) We know the Taylor series
\[ \frac{1}{1-x} = 1 + x + x^2 + \cdots. \]
We replace \( x \) with \(-x^2\) to get:

\[ \frac{1}{1+x^2} = 1 + (-x^2) + (-x^2)^2 + \cdots = 1 - x^2 + x^4 + \cdots. \]

Then we multiply by \( 2x \) and obtain:

\[ \frac{2x}{1+x^2} = 2x \cdot 1 - 2x \cdot x^2 + 2x \cdot x^4 + \cdots = 2x - 2x^3 + 2x^5 + \cdots. \]
b) We are given that
\[
\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots.
\]
From here we find:
\[
\cos x - 1 = -\frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots
\]
and
\[
\frac{\cos x - 1}{x} = -\frac{x^1}{2!} + \frac{x^3}{4!} - \frac{x^5}{6!} + \cdots.
\]
Then we integrate this term by term with respect to \(x\) from 0 to 1. Using that the anti-derivative of \(x^n\) is \(\frac{x^{n+1}}{n+1}\), we compute:
\[
\int_0^1 x^1 \, dx = \left. \frac{x^2}{2} \right|_0^1 = \frac{1^2 - 0^2}{2} = \frac{1}{2},
\]
\[
\int_0^1 x^3 \, dx = \left. \frac{x^4}{4} \right|_0^1 = \frac{1^4 - 0^4}{4} = \frac{1}{4},
\]
\[
\int_0^1 x^5 \, dx = \left. \frac{x^6}{6} \right|_0^1 = \frac{1^6 - 0^6}{6} = \frac{1}{6}.
\]
Then
\[
\int_0^1 \frac{\cos x - 1}{x} \, dx = -\frac{1/2}{2!} + \frac{1/4}{4!} - \frac{1/6}{6!} + \cdots.
\]

4. a) The double integral \(\int_R \int xe^{xy} \, dxdy\) is equal to the iterated integral
\[
\int_0^1 \int_0^2 xe^{xy} \, dy \, dx.
\]
We first compute the inner integral (the one in the parentheses). It is an integral with respect to \(y\), while \(x\) is treated as a constant. Since the derivative of \(e^{xy}\) with respect to \(y\) is \(xe^{xy}\), the anti-derivative of \(xe^{xy}\) with respect to \(y\) is \(e^{xy}\). Then
\[
\int_0^2 xe^{xy} \, dy = e^{xy} \bigg|_{y=0}^{y=2} = e^{2x} - e^0x = e^{2x} - 1.
\]
Now the outer integral becomes
\[
\int_0^1 (e^{2x} - 1) \, dx = \int_0^1 e^{2x} \, dx - \int_0^1 1 \, dx.
\]
The anti-derivative of \( e^{2x} \) is \( \frac{1}{2} e^{2x} \), while the anti-derivative of 1 is \( x \). Hence,
\[
\int_0^1 e^{2x} \, dx = \frac{1}{2} e^{2x} \bigg|_0^1 = \frac{1}{2} (e^2 - e^0) = \frac{1}{2} (e^2 - 1)
\]
and
\[
\int_0^1 1 \, dx = x \bigg|_0^1 = 1 - 0 = 1.
\]
Final answer: \( \iint_R xe^{xy} \, dxdy = \frac{1}{2} (e^2 - 1) - 1 = \frac{e^2 - 3}{2} \).

b) We first compute the inner integral. It is an integral with respect to \( y \), while \( x \) is treated as a constant. It is equal to
\[
\int_x^{x^2} xy \, dy = x \int_x^{x^2} y \, dy = x \cdot \frac{1}{2} y^2 \bigg|_{y=x^2}^{y=x} = x \cdot \frac{1}{2} ((x^2)^2 - x^2) = \frac{1}{2} (x^5 - x^3),
\]
because the anti-derivative of \( y \) is \( \frac{1}{2} y^2 \). Then we integrate this result with respect to \( x \):
\[
\int_0^1 \left( \frac{1}{2} (x^5 - x^3) \right) \, dx = \frac{1}{2} \int_0^1 x^5 \, dx - \frac{1}{2} \int_0^1 x^3 \, dx
\]
\[
= \frac{1}{2} \cdot \frac{1}{6} x^6 \bigg|_0^1 - \frac{1}{2} \cdot \frac{1}{4} x^4 \bigg|_0^1 = \frac{1}{12} (1^6 - 0^6) - \frac{1}{8} (1^4 - 0^4) = \frac{1}{12} - \frac{1}{8} = -\frac{1}{24}.
\]
Answer: \( -\frac{1}{24} \).