1. 
\[ \pm E_1 = (A \cdot 2 + B) - 2 = 2A + B - 2, \]
\[ \pm E_2 = (A \cdot 3 + B) - 0 = 3A + B, \]
\[ \pm E_3 = (A \cdot 7 + B) - (-1) = 7A + B + 1, \]
\[ E = E_1^2 + E_2^2 + E_3^2 = (2A + B - 2)^2 + (3A + B)^2 + (7A + B + 1)^2, \]
\[ \frac{\partial E}{\partial A} = 2(2A + B - 2) + 2(3A + B) + 2(7A + B + 1) = 124A + 24B + 6, \]
\[ \frac{\partial E}{\partial B} = 2(2A + B - 2) + 2(3A + B) + 2(7A + B + 1) = 24A + 6B - 2. \]

The system \( 124A + 24B + 6 = 0, \ 24A + 6B - 2 = 0 \) has a unique solution \( A = -1/2 = -0.5, \ B = 7/3 = 2.333. \) Answer: \( y = -(1/2)x + 7/3. \)

2. 
\[ \int_{\mathbb{R}} (x^2y + y^2)dx dy = \int_0^2 \left( \int_2^3 (x^2y + y^2)dy \right) dx. \]
\[ \int_2^3 (x^2y + y^2)dy = \left( x^2 \cdot \frac{y^2}{2} + \frac{y^3}{3} \right) \bigg|_{y=2}^{y=3} = x^2 \cdot \frac{3^2 - 2^2}{2} + \frac{3^3 - 2^3}{3} = \frac{5x^2}{2} + \frac{19}{3}. \]
\[ \int_0^2 \left( \frac{5x^2}{2} + \frac{19}{3} \right)dx = \left( \frac{5}{2} \cdot \frac{x^3}{3} + \frac{19}{3} \cdot x \right) \bigg|_0^2 = \frac{5}{2} \cdot \frac{2^3}{3} + \frac{19}{3} \cdot (2 - 0) = \frac{58}{3} = 19.333. \]

Answer: \( 58/3 = 19.333. \)

3. \( f(x, y) = x^3 - y^2 - 3x + 4y, \)
\[ \frac{\partial f}{\partial x} = 3x^2 - 3, \quad \frac{\partial f}{\partial y} = -2y + 4. \]

First derivative test: the critical points are the solutions of \( \frac{\partial f}{\partial x} = 0, \ \frac{\partial f}{\partial y} = 0. \)
Then \( 3x^2 - 3 = 0, \ -2y + 4 = 0 \) and \( x = \pm 1, \ y = 2. \) We find two critical points: \( (1, 2), \ (-1, 2). \)
Second derivative test:
\[
\frac{\partial^2 f}{\partial x^2} = 6x, \quad \frac{\partial^2 f}{\partial y^2} = -2, \quad \frac{\partial^2 f}{\partial x \partial y} = 0.
\]

\[
D(x, y) = \frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} - \left( \frac{\partial^2 f}{\partial x \partial y} \right)^2 = (6x)(-2) - 0^2 = -12x.
\]

\[
D(1, 2) = -12 < 0, \text{ hence no relative min/max (saddle) at } (1, 2).
\]

\[
D(-1, 2) = 12 > 0, \quad \frac{\partial^2 f}{\partial x^2}(-1, 2) = -6 < 0, \text{ hence relative max at } (-1, 2).
\]

4. \( F(x, y, \lambda) = 2x^2 + xy + y^2 + \lambda(2 - x - y) \).
\[
\frac{\partial F}{\partial x} = 0 = 4x + y - \lambda,
\]

\[
\frac{\partial F}{\partial y} = 0 = x + 2y - \lambda.
\]

\[
\lambda = 4x + y = x + 2y, \quad 3x = y.
\]

\[
0 = 2 - x - y = 2 - x - 3x, \quad x = 1/2 = .5, \quad y = 3/2 = 1.5.
\]

5. a. The first term is \( a = \frac{3}{4} = .75 \), and the ratio is \( r = \frac{-3/16}{3/4} = -\frac{1}{4} = -25 \). Since \(|r| = .25 < 1\), the series is convergent. Its sum is:
\[
\frac{a}{1 - r} = \frac{.75}{1 - (-.25)} = \frac{.75}{1.25} = .6 = \frac{3}{5}.
\]

5. b. On the first day after a dose, the patient will have \( D \) mgs of drug.
Second day: \( D + D(.3) \).
Third day: \( D + (D + D(.3))(.3) = D + D(.3) + D(.3)^2 \).
In the long run: \( D + D(.3) + D(.3)^2 + \cdots \).
This is a geometric series with \( a = D \) and \( r = .3 \). Its sum is \( \frac{a}{1 - r} = \frac{D}{1 - .3} = \frac{D}{.7} \). Then \( \frac{D}{.7} = 6, \quad D = 6(.7) = 4.2 \).
6. 
\[ f(x) = (x + 1)^{-1}, \quad f(1) = 2^{-1} = 1/2, \]
\[ f'(x) = (-1)(x + 1)^{-2}, \quad f'(1) = -2^{-2} = -1/4, \]
\[ f''(x) = (-1)(-2)(x + 1)^{-3}, \quad f''(1) = 2 \cdot 2^{-3} = 1/4, \]
\[ f'''(x) = (-1)(-2)(-3)(x + 1)^{-4}, \quad f'''(1) = -6 \cdot 2^{-4} = -3/8. \]
The third Taylor polynomial at \( x = 1 \) is:
\[ p_3(x) = f(1) + f'(1)(x - 1) + \frac{f''(1)}{2!}(x - 1)^2 + \frac{f'''(1)}{3!}(x - 1)^3 \]
\[ = \frac{1}{2} - \frac{1}{4}(x - 1) + \frac{1}{8}(x - 1)^2 - \frac{1}{16}(x - 1)^3. \]

7. Differentiate
\[ \frac{1}{1 - x} = 1 + x + x^2 + x^3 + \cdots, \]
to get
\[ \frac{1}{(1 - x)^2} = 1 + 2x + 3x^2 + \cdots, \]
then replace \( x \) with \( -x \) and obtain
\[ \frac{1}{(1 + x)^2} = 1 - 2x + 3x^2 + \cdots. \]

8. Note that \( y = 0 \) is a constant solution, but it doesn’t satisfy the initial condition \( y(0) = 1 \). Hence, we can assume \( y \neq 0 \) and we can divide by \( y^3 \).
\[ y' = y^3t + y^3, \quad \frac{y'}{y^3} = t + 1, \quad y^{-3}y' = t + 1, \]
\[ \int y^{-3}y'dt = \int (t + 1)dt + C, \quad \int y^{-3}dy = \int (t + 1)dt + C, \]
\[ \frac{y^{-3+1}}{-3+1} = \frac{t^2}{2} + t + C, \quad y^{-2} = -t^2 - 2t - 2C, \quad y^{-2} = \frac{1}{-t^2 - 2t - 2C}, \]
\[ y = \pm \frac{1}{\sqrt{-t^2 - 2t - 2C}}. \]
Since \( y(0) = 1 > 0 \), our solution is with + sign. Then \( y(0) = 1 \) gives \( \frac{1}{\sqrt{-2C}} = 1 \), \(-2C = 1\).

Answer: \( y(t) = \frac{1}{\sqrt{-t^2 - 2t + 1}} \).

9. \( y(t) = Ce^{-\lambda t}, \quad y(0) = 5, \quad y(6) = 3, \)
\[ Ce^{-\lambda 0} = C = 5, \quad Ce^{-\lambda 6} = 5e^{-6\lambda} = 3, \quad e^{-6\lambda} = 3/5 = .6, \]
\[-6\lambda = \ln .6, \quad \lambda = -\frac{\ln .6}{6} \approx .0851.\]

Solve for \( t \) such that \( y(t) = 1 \). Then \( Ce^{-\lambda t} = 5e^{-\lambda t} = 1, \quad e^{-\lambda t} = 1/5 = .2, \)
\[-\lambda t = \ln .2, \quad t = \frac{\ln .2}{\lambda} = \frac{6\ln .2}{\ln .6} \approx 18.9 \approx 19.\]

Answer: 19 days.

10. \( z = (y + 1)(y - 7) \) is a concave up parabola with zeroes \( y = -1 \) and \( y = 7 \) and vertex at \( y = (-1 + 7)/2 = 3 \).

The constant solutions to \( y' = (y + 1)(y - 7) \) are: \( y = -1 \) and \( y = 7 \).

The solution with initial condition \( y(0) = -2 \) is increasing, concave down, and asymptotic to \( y = -1 \).

The solution with initial condition \( y(0) = 3 \) is decreasing, concave up, and asymptotic to \( y = -1 \).

The solution with initial condition \( y(0) = 10 \) is increasing, concave up, and unbounded.

11. a. \( y' = .03y - M, \quad y(0) = 500,000. \)

11. b. The zero of \( .03y - M \) is \( y = M/.03 \). When \( 500,000 = M/.03 \), the solution \( y(t) \) is constant. When \( 500,000 > M/.03 \), it is increasing, and when \( 500,000 < M/.03 \), it is decreasing. We don’t want \( y(t) \) to be decreasing, because then it will reach zero. Therefore, \( 500,000 \geq M/.03 \), which gives \( M \leq 500,000 \times .03 = 15,000 \). Answer: 15,000.