Copula-Based Nonlinear Models of Spatial Market Linkages *

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November 1, 2011

Abstract

An extensive empirical literature has addressed a wide array of issues pertaining to price linkages over space and across time. Empirical models of price linkages have been used to measure market power and to characterize the operation of markets that are separated by space, time, and product form. The long history of these empirical models extends from simple tests of price correlation, to conventional regression tests, to modern time series models that account for nonstationarity, nonlinearities, and threshold behavior in market linkages. This paper proposes an alternative and potentially novel approach to analyzing these same types of time series data in a nonlinear fashion. Copula-based models that consider the joint distribution of prices separated by space are developed and applied to weekly prices for important construction materials produced in geographically distinct markets. In particular, we consider prices taken from weekly editions of the *Random Lengths* publication for homogeneous oriented strand board products.

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1 Introduction

Notions of price parity, spatial arbitrage, and price transmission characterize many basic principles and relationships in economics. At the core, markets should efficiently function so as to eliminate any potential for riskless profits through arbitrage and trade. This fundamental condition is often called the "Law of One Price" (LOP)—a concept whose nomenclature reflects the considerable confidence that economists place in its adherence. Over the years there has been considerable interest in and debate about the empirical validity of the Law of One Price (LOP), especially as it pertains to markets for tradeable goods. On one hand, economists take it as being nearly axiomatic that freely functioning markets for traded, homogeneous products should ensure that prices are efficiently linked across regional markets, the implication being that no persistent opportunities for spatial arbitrage profits exist.[1] The general implication underlying these basic concepts is that prices for homogeneous products at different geographic locations in otherwise freely functioning markets should differ by no more than transport and transactions costs, the latter including, for example, insurance, contracting fees, licensing fees, legal fees, and possibly a risk premium. On the other hand, there is substantial empirical evidence in a huge literature that finds that the adjustment lags required to restore arbitrage equilibria are often found to be far longer than would seem natural based upon any reasonable understanding of the mechanics of physical trade as it pertains to the markets in question.

A related avenue of research considers price transmission in a more general sense. Here the focus is typically on how shocks or changes in market conditions at one location or level of the market are transmitted to other locations or market levels. Price transmission models

[1] Distinctions between tests of LOP the and spatial market integration are not especially meaningful. In both cases, the economic phenomena being evaluated (spatial market arbitrage) is identical. A survey of both strands of literature can be found in Fackler and Goodwin (2001).

are often applied in considerations of vertical market linkages. For example, the extent to which raw commodity markets are impacted by changes at the retail level is an issue that has received considerable attention in the empirical literature. Although the economic phenomena being evaluated in these studies may be different, the empirical tools used to evaluate such market linkages are often identical to those used in evaluating spatial market linkages.

Early empirical studies generally fail to find support in favor of LOP. Isard (1977) finds rather conclusive evidence against the LOP using disaggregate data for traded goods. Isard's conclusions were subsequently confirmed for a variety of commodities in a wide array of market settings by, among others, Richardson (1978), Thursby, Johnson, and Grennes (1986), Benninga and Protopapadakis (1988), and Giovannini (1988). Goodwin, Grennes, and Wohlgenant (1990) do, however, find some support for the LOP when it was specified in terms of price expectations as opposed to observed prices. Following Engle and Granger's seminal paper (1987), cointegration techniques have been widely used to rationalize the LOP as a long–run concept. By adopting this view of the LOP, economists have been able to find more compelling evidence in favor of the LOP, including, for example, Buongiorno and Uusivuori (1992) (U.S. pulp and paper exports), Michael, Nobay, and Peel (1994) (international wheat prices), Bessler and Fuller (1993) (U.S. regional wheat markets), and Jung and Doroodian (1994) (softwood lumber markets).

The most recent literature in this area applies smooth or discrete threshold time series models that typically consider refinements of autoregressive or vector error correction models in analyzing price relationships. The underlying motivation is that adjustments to equilibrium may not be linear, and that this nonlinearity may, in turn, be associated with hard–to–observe transactions costs associated with arbitrage. The theoretical underpinnings for transactions–costs–induced nonlinearity in the LOP are put forward by Dumas (1992), although the basic idea dates back at least to the work of Heckscher (1916), who notes that transactions costs may define "commodity points" within which prices are not directly linked because the price differences are less than the costs of trade.

A recent example includes an analysis of manufactured lumber products (oriented strand board or OSB) in the U.S. undertaken by Holt, Prestemon, and Goodwin (2011). Their analysis applies smooth transition vector autoregression (STAR) models to consider price relationships among spatially-distinct North American markets for manufactured OSB. The application is notable in light of recent litigation that charged that OSB manufacturers had practiced discriminatory and noncompetitive pricing during the latter part of the decade. The analysis reveals that nonlinearity is an important feature of price relationships in these markets and that the price parity relationships implied by economic theory and efficient arbitrage are generally supported by the STAR models. Other empirical investigations of the role of nonlinearity as pertains to the LOP are reported by Goodwin and Piggott (2001), Lo and Zivot (2001), Sephton (2003), Balcombe, Bailey, and Brooks (2007), and Park, Mjelde, and Bessler (2007). In general, these studies find support for threshold effects, with the path of adjustment to equilibrium depending typically on the size if not the sign of the shock. In particular, large shocks that lead to profitable arbitrage opportunities net of transactions costs are quickly eliminated whereas smaller shocks, which may not be large enough to result in profitable arbitrage opportunities, may elicit a much smaller effect or even no adjustment at all.

This extensive literature has several common themes and generally involves the application of conventional time series models to finely sampled, nonstationary price data. In this paper, we propose an alternative and potentially novel approach to analyzing these same types of time series data in a nonlinear fashion. We develop copula-based models that consider the joint distribution of prices separated by space and apply them to weekly prices for homogeneous OSB products at geographically–distinct North American markets. Although copula models have been extensively used in financial economics and risk management studies they have not been extensively applied in modeling nonlinear, spatial arbitrage relationships.[2]

[2]Patton (2006) allows for time variation in the conditional joint distribution of the returns on the Deutsche mark/U.S. dollar and Japanese Yen/U.S. dollar exchange rates by allowing the parameter(s) of a given copula to vary through time. Smith et al. (2011) examine linkages among logarithmic prices in regional Australian electricity markets but do not explicitly model the nonlinear error-correction adjustment process

Our approach is a natural extension of the existing (and abundant) time-series evaluations of spatial price linkages. It involves direct examination of the joint probability distribution of the key economic variables of interest. In this way, the approach is really no different than standard maximum likelihood methods applied to structural or non-structural econometric models. However, we give particular attention to the nature of the jointness or correlation between these key variables. In particular, we allow this correlation to be "state–dependent" and therefore to depend upon market conditions at any particular point in time. In this manner, our approach is analogous to the regime-switching and threshold models that are frequently applied in evaluating spatial and vertical market linkages.

The plan of our paper is as follows. The next section outlines conventional empirical approaches typically used to evaluate spatial price linkages. We then propose an alternative approach that is based upon copula models of the joint likelihood function of price differentials and first-differences of price differentials. The third section presents an empirical application of these models to an important, regionally-traded, homogeneous commodity market—the North American Oriented Strand Board (OSB) market. In particular, we consider price linkages among four regionally-separated OSB markets. OSB is a commodity of interest because it has become one of the leading building materials used in the construction sector of the U.S. and in many other countries. OSB surpassed plywood as the leading engineered wood product in the mid-1990s in the U.S. The final section contains a summary of the results and conclusions.

2 Econometric Models of Spatial Price Relationships

As we have noted, a vast empirical literature has considered a wide array of empirical models of price relationships across space, time, and market form. This literature has evolved from

that is standard in spatial arbitrage models and that we consider here. Reboredo (2011) also examines the comovement of crude oil prices using copulas, though his approach is also fundamentally different from that considered here in that it does not directly consider the error–correction process commonly applied in models of spatial price linkages.

a simple consideration of correlation coefficients and linear regression models to regime-switching, time-series models that allow for a form of "state-dependence" when characterizing price linkages. The most recent literature is usually based upon a standard autoregressive model of the form:

$$\Delta(p_t^i - p_t^j) = a + b(p_{t-1}^i - p_{t-1}^j) \tag{1}$$

where p^i and p^j are logarithmic prices in regions i and j, respectively, and a and b are parameters that reflect the degree of market integration.[3] In particular, b represents the degree of "error-correction" that characterizes departures from price parity, which are reflected in large values of $p_{t-1}^i - p_{t-1}^j$. The "error" term, represents proportional deviations from market equilibrium. In some cases, a is taken to represent a proportional price difference that reflects transactions costs.[4]

Recent empirical evaluations of spatial price linkages have recognized that the presence of transactions costs, which are notoriously difficult to measure but nonetheless are likely to be relevant in any empirical consideration of spatial commodity trade, may result in nonlinearities in the estimates of equation 1. Two specific avenues have been adopted to account for such nonlinearities. In the first, a "threshold" parameter that reflects the presence of transactions costs is estimated. The linkage between prices varies depending upon whether the departure from equilibrium represented by $p_{t-1}^i - p_{t-1}^j$ is large enough to evoke spatial arbitrage. In this case, a discrete break occurs between regimes where one regime may represent a case of no–trade while another represents conditions of profitable trade and arbitrage. These models are typically referred to as "threshold autoregressive" (TAR) or "threshold vector error-correction" (TVEC) models.

Alternatives to this simple model permit the switching between regimes to occur at a gradual and smooth pace. The speed and degree of adjustment is implied by parameters of

[3]See, for example, Taylor (2001), who applies regime–switching, time–series models of this form to empirical tests of purchasing power parity—an aggregate version of the LOP.

[4]A specification that is often referred to as an "iceberg" model, reflecting the fact that the value of the commodity melts away via a proportionally lower price as it is shipped.

a "transition" function. A number of different specifications of such "smooth transition autoregressive " (STAR) models have been developed in the literature. Such models essentially nest the TAR versions such that they permit a more flexible evaluation of price linkages. The behavior underlying spatial price linkages is likely to be discrete—representing the two states of trade/no-trade. However, in that empirical evaluations of such models almost always involves some degree of aggregation, the patterns of adjustment may be of a more smooth nature and therefore may favor the STAR-type models.

These models have provided considerable flexibility in modeling spatial and vertical price linkages. The results of allowing for such flexibility and accounting for unobservable transactions costs have generally provided much greater support for the concept of market integration and efficiency. However, in empirical practice, they often suffer from complications resulting from parameters that may be unidentified under certain null hypotheses and a resulting need to rely upon non–standard inferential techniques.

Our approach involves a simple extension or re–characterization of the fundamental relationship expressed in equation 1. We make use of the widely-recognized correspondence between β in equation 1 and the standard, linear Pearson correlation coefficient:

$$\hat{\beta} = \hat{\rho}\frac{\hat{\sigma_y}}{\hat{\sigma_x}} \tag{2}$$

where y and x correspond to the random variables $\Delta(p_t^i - p_t^j)$ and $p_{t-1}^i - p_{t-1}^j$, ρ is the Pearson correlation coefficient, and σ_y (σ_x) represents the standard deviation of random variable y (x). The "error-correction" relationship that characterizes the linkage between markets i and j is represented in the sample correlation coefficient ρ. To the extent that b realizes regime switching, the coefficient ρ will also reflect switching. To the extent that such switching is dependent upon market conditions (i.e., as reflected in the price differential), the correlation coefficient ρ may exhibit state–dependence.

The empirical approach adopted here involves considering the joint distribution function of $\Delta(p_t^i - p_t^j)$ and $p_{t-1}^i - p_{t-1}^j$. We make use of a widely-recognized, fundamental result known as Sklar's (1959) Theorem, which implies that any joint probability function can be represented in terms of the marginal densities and a function known as a "copula." In particular, Sklar's Theorem implies that, for any continuous $p-$variate cumulative probability function F, a unique copula function $C(\cdot)$ exists for which

$$F(x_1, x_2, ..., x_p) = C(F_1(x_1), ..., F_p(x_p); \xi) \tag{3}$$

where $F_i(\cdot)$ are marginal distributions and ξ is a set of parameters that measures dependence.

2.1 Copulas

Copula models have recently realized widespread application in empirical models of joint probability distributions.[5] The models essentially use a "copula" function to tie together two marginal probability functions that may (or may not) be related to one another. Much of the work on copulas has been motivated by their applicability to the issues in risk management, insurance and financial economics (see, among others, Rodriguez (2003), Cherubini et al. (2004), Hu (2006), Patton (2006), and Jondeau and Rockinger (2006)). In the empirical literature, copula models have been used extensively in the design and rating of crop revenue insurance contracts, where the inverse correlation of prices and yields plays an important role in pricing revenue risk.

A p–dimensional copula, $C(u_1, u_2, \ldots, u_p)$, is a multivariate distribution function in the unit hypercube $[0, 1]^p$ with uniform $U(0, 1)$ marginal distributions. As long as the marginal distributions are continuous, a unique copula is associated with the joint distribution, F, that can be obtained as:

$$C(u_1, u_2, \ldots u_p) = F(F_1^{-1}(u_1), \ldots, F_p^{-1}(u_p)). \tag{4}$$

In a similar fashion, given a p–dimensional copula, $C(u_1, \ldots u_p)$, and p univariate distributions, $F_1(x_1), \ldots, F_p(x_p)$, the equation 3 is a p–variate distribution function with marginals $F_1, \ldots F_p$ whose corresponding density function can be written as:

$$f(x_1, x_2, \ldots x_p) = c(F_1(x_1), \ldots, F_p(x_p)) \prod_{i=1}^{p} f_i(x_i) \tag{5}$$

[5]For details on construction and properties of copulas, see among others, Joe (1997) and Nelsen (2006).

Provided that it exists, the density function of the copula, c, can be derived using equation 4 and marginal density functions, f_i:

$$c(u_1, u_2, \dots u_p) = \frac{f(F_1^{-1}(u_1), \dots, F_p^{-1}(u_p))}{\prod_{i=1}^{p} f_i(F_i^{-1}(u_i))}$$

There is a large number of parametric families of copulas applied in the literature. Two of the most commonly used copula families are elliptical copulas and Archimedean copulas. Gaussian and t–copulas are examples of elliptical copulas while the Clayton and Gumbel are among Archimedean copulas.

The Gaussian (or normal) copula, which is obtained from the multivariate normal distribution with correlation matrix R, is the most commonly applied copula. It can be written as:

$$C_R^{Ga}(u_1, u_2, \dots, u_p) = \int_{-\infty}^{\Phi^{-1}(u_1)} \dots \int_{-\infty}^{\Phi^{-1}(u_p)} \frac{1}{\sqrt{2\pi^p |R|}} \times exp\left\{\frac{-\mathbf{u}' R^{-1}\mathbf{u}}{2}\right\} d\mathbf{u} \tag{6}$$

where $\mathbf{u} = (u_1, \dots, u_p)$ and Φ^{-1} is the inverse of the cumulative distribution function $(c.d.f)$ of the univariate standard normal distribution.

The Gaussian copula assumes that there is no dependence in the tails of the distribution. A more flexible elliptical copula can be derived from the multivariate t-distribution. The $t-$copula, for a multivariate $t-$distribution with ν degrees of freedom and correlation matrix, R, is given by:

$$C_{\nu,R}^{t}(u_1, u_2, \dots, u_p) = \int_{-\infty}^{t_\nu^{-1}(u_1)} \dots \int_{-\infty}^{t_\nu^{-1}(u_p)} \frac{\Gamma(\frac{\nu+p}{2})(1 + \frac{\mathbf{u}' R^{-1}\mathbf{u}}{\nu})^{-\frac{\nu+p}{2}}}{\Gamma(\frac{\nu}{2})\sqrt{(\pi\nu)^p |R|}} d\mathbf{u} \tag{7}$$

where $t_\nu^{-1}(u_1)$ denotes the inverse of the cumulative distribution function of the standard univariate t–distribution with ν degrees of freedom. Note that the Gaussian copula is a special case of the t–copula when ν approaches infinity. The properties of t–copula have been studied by Embrechts et al. (2002), Fang et al. (2002), and Demarta and McNeil (2005). The t–copula model has received much attention recently, particularly in the context of modeling multivariate financial data (e.g., daily relative or logarithmic price changes on a number of stocks). One reason for the popularity of the t–copula in these applications is its ability to capture the phenomenon of dependent extreme values. The dependence in elliptical distributions is essentially determined by covariances.[6]

A different parametric family is given by the Archimedean class of copulas. If we define a function $\phi : [0,1] \to [0,\infty)$ to be a strict function that satisfies $\phi(0) = \infty$ and $\phi(1) = 0$ and has an inverse, ϕ^{-1}, that is monotonic on $[0,\infty)$, then an Archimedean copula can be defined as:

$$C(u_1, u_2, \ldots, u_p) = \phi^{-1}\Big(\phi(u_1) + \cdots + \phi(u_p)\Big)$$

$\phi : [0,1] \to [0,\infty)$ is known as an *Archimedean copula generator* function. Among Archimedean copulas, we consider the Clayton and Gumbel copula functions.[7]

Specific Archimedean copulas are defined by their generator functions. For a generator function of $\phi(u) = \theta^{-1}\left(u^{-\theta} - 1\right)$, a Clayton copula is given by:

$$C_\theta^C(u_1, u_2, \ldots u_p) = \left[\sum_{i=1}^{p} u_i^{-\theta} - p + 1\right]^{-1/\theta} \tag{8}$$

with $\theta > 0$. Alternatively, for the generator function $\phi(u) = (-\log(u))^\theta$, a Gumbel copula is defined as:

$$C_\theta^{Gu}(u_1, u_2, \ldots, u_p) = \exp\left\{-\left[\sum_{i=1}^{p}(-\log u_i)^\theta\right]^{1/\theta}\right\} \tag{9}$$

with $\theta > 1$. Relationships among random variables are typically characterized by various measures of correlation or dependence. The advantage of many copula functions lies in their ability to allow for varying degrees of correlation or dependence in the bivariate distribution. Correlation is typically measured by parametric correlation coefficients, such as the linear Pearson correlation coefficient, or by various nonparametric, rank-type correlation measures, such as Kendall's *tau*. Various dependence measures between two random variables, X_1 and X_2, depend only on their copula function.

In the case of spatial price relationships, the degree of dependence that characterizes relationships in the tails of the distribution is of particular relevance. The notion of smooth

[6]See Embrechts et al. (2002) and Glasserman (2004) for detailed discussions on the application of t–copulas in risk management.

[7]We also consider rotated versions of each of these copula functions. A copula is rotated by using $1 - u_i^x$ in place of u_i^x, where u_i^x is the quantile corresponding to the marginal distribution for x_i.

or discrete regime switching models that are commonly used in empirical studies of spatial arbitrage is that the relationships among price differentials may be different when such differentials are large. Large price differentials correspond to large deviations from equilibrium arbitrage conditions and should therefore correspond to faster rates of adjustment to return markets to equilibrium conditions. The fact that such dependence may be different for large price differentials reflects the influences of unobservable transactions costs. In particular, price differentials that do not exceed transactions costs do not imply profitable arbitrage opportunities while large differentials in excess of transactions costs should be eliminated quickly through arbitrage behavior. Copula models are especially well-suited to considering tail behavior in that they allow for more flexible characterizations of tail dependence. The coefficients of upper tail dependence, λ_U, and lower tail dependence, λ_L, are defined as:

$$\lambda_U = \lim_{u \to 1} P(X_2 > F_{X_2}^{-1}(u) | X_1 > F_{X_1}^{-1}(u)) \tag{10}$$

$$\lambda_L = \lim_{u \to 0} P(X_2 \leq F_{X_2}^{-1}(u) | X_1 \leq F_{X_1}^{-1}(u)) \tag{11}$$

These coefficients of tail dependence, λ_U and λ_L, can be expressed as a function of a copula as:

$$\lambda_U = \lim_{u \to 1} \frac{1 - 2u + C(u, u)}{1 - u} \tag{12}$$

and

$$\lambda_L = \lim_{u \to 0} \frac{C(u, u)}{u} \tag{13}$$

Different copulas allow for differing degrees of tail dependence. The Gaussian copula is characterized by zero tail dependence. The $t-$copula exhibits symmetric tail dependence which is determined by:

$$\lambda_U = \lambda_L = 2t_{\upsilon+1}\left(\frac{-\sqrt{\upsilon+1}\sqrt{1-\rho}}{\sqrt{1+\rho}}\right)$$

where $t_{\upsilon+1}$ denotes the cumulative distribution function of the standard univariate Student–t distribution with $\upsilon+1$ degrees of freedom. The Clayton copula has zero upper tail dependence and an lower tail dependence defined by:

$$\lambda_L = 2^{(-1/\theta)} \tag{14}$$

where θ is a parameter of the copula, $\theta > 0$. The copula parameter θ is related to Kendall's *tau*, τ, by $\theta = \frac{2\tau}{1-\tau}$. The rotated version has zero upper tail dependence and an equivalent expression for lower tail dependence. The Gumbel copula has zero lower tail dependence and an upper tail dependence given by:

$$\lambda_U = 2 - 2^{(1/\theta)} \tag{15}$$

For the Gumbel copula, θ is related to Kendall's *tau* by $\theta = (1-\tau)^{-1}$.

3 Empirical Application

Consider a homogeneous commodity traded in two regional markets represented, respectively, by location indices i and j. The regional market prices are denoted by P_i and P_j. The per–unit revenue to arbitragers selling in region j is therefore $(1-\kappa)P_j$, where κ denotes the per–unit proportional loss in value for the commodity due to transactions (or transport) costs, $0 < \kappa < 1$. In general, the greater the distance between locations i and j, the closer is κ to one. A simple model of spatial price relationships that incorporates the effects of transactions costs (and possibly other frictions), then, can be written as:

$$1/(1-\kappa) \geq P_i/P_j \geq (1-\kappa) \tag{16}$$

or, after taking natural logarithms and denoting $p_i = ln(P_i)$ and $p_j = ln(P_j)$,

$$-ln(1-\kappa) \geq (p_i - p_j) \geq ln(1-\kappa) \tag{17}$$

The implication from equation 17 is that there is a band, $[-ln(1-\kappa), ln(1-\kappa)]$, within which no profitable arbitrage activity will occur; arbitrage is, however, profitable when log price differentials, $p_i - p_j$, fall outside of the limits of the band. Over time we would expect that log price differentials within the limits of the band would follow something very close to a unit root process, likely without drift. However, log price differences that fall outside of the limits of the band should be mean–reverting. The relationship in equation 17 implies a transactions cost band, which has often been assumed in the literature, and which typically yields an empirical model consistent with the threshold autoregressive models described above (see, for example, Goodwin and Piggott (2001) and Balcombe, Bailey, and Brooks (2007)). As noted, these models typically find that market price adjustments to shocks to parity condition tend to be faster or more apparent when the shocks are large. Threshold models typically allow the speed or degree of adjustment to vary in accordance with the size of the disequilibrium implied in parity relationships. In particular, threshold models usually recognize the fact that small price differences may not exceed the costs of conducting spatial trade and arbitrage whereas large differences may imply arbitrage opportunities that are more quickly eliminated. In this analysis, we use copula–based models as an alternative nonlinear model that captures much of the same behavior addressed in nonlinear threshold and smooth transition models.

The copula approach offers an alternative approach to representing the multivariate distribution in terms of its (possibly) dependent marginals. This may be accomplished by following a variety of estimation approaches, including conventional joint maximum likelihood estimation or by following a two-stage statistical procedure that separately estimates the marginal distributions and the copula function. In this analysis, we choose nonparametric (empirical) *c.d.f.*'s so as to allow for maximum flexibility. That said, properties of individual marginal distributions may be of interest in their own right. Such properties can be discerned the applying maximum likelihood (ML) or method of moments estimation techniques in conjunction with or preceding ML estimation of the joint distribution (i.e., by way of the copula). The two–step approach that we follow here is often termed the "Canonical Maximum Likelihood" method.

In the case of the joint probability distribution among spatially linked price pairs, we fit a copula model for the joint distribution of $\Delta(p_t^i - p_t^j)$ and $(p_{t-1}^i - p_{t-1}^j)$, which corresponds to the relationship in equation 1. Depending on the specific copula model applied, differing degrees of nonlinearity in the relationship among the first differences and lagged levels of price differentials may be implied. Specifically, the particular choice of the copula function determines the nature of correlation. Standard linear correlation generally implies a constant correlation coefficient. In contrast, different functional relationships between random variables, including those that vary across the marginals, can be achieved with copula functions. In particular, the parametric form of the copula can, in some cases, permit considerable flexibility in how adjustments may differ as price differences become larger or smaller. In our application, we consider six different copula models (Gaussian, t, Clayton, rotated Clayton, Gumbel, and rotated Gumbel) which allow for varying degrees of tail and state dependence as the degrees of freedom parameter changes. In a t–copula, for example, a smaller degrees of freedom parameter (which we denote as ν) will imply a greater degree of tail dependence. Conversely, as the degrees of freedom parameter increases, the t–copula approaches a Gaussian copula and tail dependence, therefore, approaches zero.

As an alternative to the standard copula model, we also consider a second and more deliberate approach to allowing for state–dependence in the joint distribution of regional prices. We accomplish this by allowing one or more parameters of the copula function to vary as market conditions change. We adopt an approach that is very similar to that applied in standard nonlinear threshold and smooth transition models of prices in that we allow lagged price differences (analogous to an error correction term) to directly impact the parameters of copulas that characterize the relationship between price pairs in the regional markets. For Gaussian and t–copulas, we allow the off–diagonal element of the correlation matrix R in equations 6 and 7 above to be a function of market conditions, reflected in the lagged price differentials, as follows:

$$\rho_t = 2\left(\frac{1}{exp(-(\alpha_0 + \alpha_1\varphi(p^i_{t-1} - p^j_{t-1}))}\right) - 1 \tag{18}$$

where α_0 and α_1 are parameters to be estimated. Note that this specific functional form restricts the correlation coefficient ρ to lie between -1 and 1 and φ is the empirical $c.d.f$ which implies various *states* of regional price differences.[8] In a similar fashion, the degrees

[8]We considered a number of such "forcing variables"—variables that force the change in correlation or dependence. In the end, the empirical $c.d.f$ of the price differential, which represents a normalized measure of the size of deviations from parity, yielded the best results among the alternatives considered. The optimal choice of a forcing variable remains a topic of ongoing research.

of freedom parameter in the t–copula is also allowed to vary in the following manner:

$$\nu_t = 1 + exp(\beta_0 + \beta_1 \varphi(p^i_{t-1} - p^j_{t-1})) \tag{19}$$

In the Clayton copula, we allow the shape parameter, θ, to vary using the following functional relationship for the shape parameter θ:

$$\theta_t = exp(\gamma_0 + \gamma_1 \varphi(p^i_{t-1} - p^j_{t-1})). \tag{20}$$

Likewise, for the Gumbel copulas, we allow the shape parameter to vary according to the following functional relationship:

$$\theta_t = 1 + exp(\gamma_0 + \gamma_1 \varphi(p^i_{t-1} - p^j_{t-1})). \tag{21}$$

These specifications ensure that the parameters are restricted to lie within the appropriate parameter spaces.

In accordance with the conventional error correction behavior anticipated in spatially integrated markets, we expect to see the parameters α_1, β_1 and γ_1 to be statistically significant, at least to the extent that standard copula models are not flexible enough to adequately represent the dependence structures implied by the fixed coefficients. In the case of asymmetric copulas, a parametric structure that only allows tail dependence in one direction may be implied. This is justified if trade tends to mostly be unidirectional, as is typical in most regional commodity market relationships. In such cases, depending on the direction of trade flows and which price is usually higher, the sign of these parameters could be negative or positive. This reflects the fact that an increase in the higher price or a decrease in the lower price will trigger a tighter relationship between the two prices (in first-differenced form) in the subsequent period, and assumes that markets display a relatively stable basis relationship, such that one price is generally above another (a characteristic that exists in most regional markets, where one market is usually “upstream” and the other is “downstream”). We estimate the parameters of the copula models using maximum likelihood techniques.

3.1 Data

For the empirical analyses, we consider regional North American markets for a prominent traded commodity—oriented strand board (OSB). OSB is a manufactured wood product that was first introduced in 1978 (the forerunner to oriented strand board was waferboard).[9] The Structural Board Association (SBA) reports that in 1980 OSB panel production in North America was 751 million square feet (on a $3/8^{th}$'s inch basis); however that by as early as 2005, this number had grown to 25 billion square feet. The SBA also reports that by 2000 OSB production exceeded that of plywood, and that by 2006 OSB production enjoyed a sixty–percent market share among all panel products in North America. OSB now accounts for the largest share of the overall panel wood products market.

Spatial linkages in the OSB market are of particular interest because it is a good that is widely traded across considerable distances within the North American continent. Consumption is widespread and spatially dispersed while production tends to be concentrated in particular regions such as the U.S. South and Eastern Canada. Depletion of old–growth timber stocks that traditionally served as a source for panel wood products brought about tremendous growth in the use of engineered wood products such as OSB. A burgeoning housing market (and its more recent contraction) have brought about a number of significant shocks to this rapidly expanding industry. Construction market responses to large hurricanes such as Andrew in 1992 and Katrina in 2005 are another source of OSB market price volatility that merits clearer understanding for better quantifying the economic impacts of these catastrophic events. These and related factors underscore the importance of understanding and quantitatively measuring linkages among regional OSB markets.

The data set consists of OSB in four regional North American markets. Specifically, the regions examined are: (1) Eastern Canada (production deriving from plants in Ontario and Quebec); (2) North Central U.S. (production deriving from plants in Wisconsin, Michigan,

[9] OSB is engineered by using waterproof and heat cured resins and waxes, and consists of rectangular shaped wood strands that are arranged in oriented layers. OSB is produced in long, continuous mats which are then cut into panels of varying sizes. In this regard OSB is similar to plywood, although OSB is generally considered to have more uniformity than plywood and is, moreover, cheaper to produce.

and Minnesota); (3) Southeast U.S. (production deriving from plants in Georgia, Alabama, Mississippi, South Carolina, and Tennessee); and (4) Southwest U.S. (production deriving from plants in Texas, Louisiana, Arkansas, and Oklahoma). The result is that there are six pairwise spatial price relationships that may be examined. The price data are for panels of $7/16^{th}$'s inch thickness, and are expressed in U.S. dollars per thousand square feet. All price data are observed on a weekly basis and were obtained from the industry source *Random Lengths*.[10] The data span the period from February 3, 1995 through August 20, 2010, which yields 812 weekly observations. The basic unit of analysis used throughout the analysis is the natural logarithm of the price ratio, that is, $ln(P_t^i/P_t^j)$, where i and j indicate regional locations (i.e., $i, j = 1, \ldots, 4$) and a subscripted t is a time index such that $t = 1, \ldots, T$, where $T = 812$.

3.2 Results

Our initial empirical analysis begins with a consideration of the relationship between the first-difference of the price differential, $\Delta(p_t^i - p_t^j)$, and the lagged price differential, $p_{t-1}^i - p_{t-1}^j$. Because certain Archimedean copula functional relationships are only able to accommodate positive correlation, we utilize the negative value of the price differential (or, $p_{t-1}^j - p_{t-1}^i$) as the right hand side regressor. Figure 1 illustrates the sample data for each of the six market pair combinations. The anticipated positive correlation is apparent for each market pair, suggesting adherence to the conditions required for spatial market integration. We apply OLS estimation techniques to the standard error-correction specification presented in equation 1 above (with the sign of the lagged price differential reversed). Parameter estimates and summary statistics are presented in Table 1. The results indicate a reasonably strong degree of integration among the regional OSB markets. In fact, the half–lives of deviations

[10] *Random Lengths* is an independent, privately owned price reporting service, providing information on commonly produced and consumed wood products in the U.S., Canada, and other countries since 1944. Reported open–market sales prices are based on hundreds of weekly telephone interviews with producers, wholesalers, distributors, secondary manufacturers, buying groups, treaters, and some large retailers. The regional OSB price data used are FOB mill price averages.

from equilibrium conditions implied by these estimates are very similar to those presented by Goodwin et al. (2011) in an application of STAR models to a similar set of OSB data.[11]

Marginal cumulative distributions for $\Delta(p_t^i - p_t^j)$ and $p_{t-1}^i - p_{t-1}^j$ are represented using nonparametric, empirical *c.d.f.*'s. This approach affords maximum flexibility in evaluating the functional relationships underlying market linkages. Standard maximum likelihood estimation techniques are used to fit the six different copula models described above to the resulting data. Table 2 presents ML estimates of the copula parameters and summary statistics. In particular, values of the log-likelihood functions and of the AIC model fitting criterion are presented for each copula/market–pair combination. Although these values can be compared within a parametric family, they are not be fully comparable across families. Genest et al. (2009) discuss a number of goodness of fit statistics based upon a general Cramér von Mises type of statistic. These tests are based on a comparison of the nonparametric copula function and the proposed parametric copula. Deheuvels (1979) introduces the idea of rank-based, nonparametric empirical copula functions and demonstrates that they converge uniformly to the underlying true parametric copula. We evaluate the cumulative squared differences of fitted values of each parametric copula, based on the empirical data, and the corresponding empirical copula at percentiles ranging from 5% to 95% in 5% increments. The resulting test statistic is given by:

$$\text{CvM} = \sum_{j=1}^{J} (C_j^E - \hat{C}_j^{\,P})^2 \tag{22}$$

where C_j^E is the empirical copula estimate and $\hat{C}_j^{\,P}$ is the fitted parametric copula, based on the data. The test statistic can be compared to standard Cramér von Mises tabulated critical values and can also be compared across different copula models to select the specification yielding the best fit.[12] The distributions across quantiles underlying the CvM statistic are presented in Figure 3. The models all show a high degree of correspondence to the empirical

[11] Deviation half–lives represent the weeks required to eliminate one–half of the deviation from equilibrium and are given by $ln(0.5)/ln(1-\beta)$.

[12] Genest et al. (2009) note that the limiting distribution of the statistic may be sensitive to sample size and to the family of copulas included under the composite null hypothesis. They recommend a double bootstrap procedure as an alternative to tabular critical values. In that we work with a relatively large sample and the tests statistics are very small relative to tabular critical values, we do not pursue explicit specification testing using a bootstrap but rather use the statistics to compare the goodness of fit of alternative copulas.

quantiles across the distribution. The notable differences that occur around the medians reflect a concentration of identical observations, which correspond to a situation of static price differentials.

Likewise, measures of tail dependence, as described above, are also presented in Table 2. The correlation, degrees of freedom (in the case of the t–copula), and shape parameters are all highly statistically significant. Recall that a t–distribution converges to a Gaussian distribution as the degrees of freedom increases. In four of the six cases, the degrees of freedom parameters for the t–copula are less than 30, which indicates greater platykurtosis than would be suggested by a Gaussian copula.

Tail dependence is an important indicator of how the relationship between the variables of interest (price differentials) behaves under extreme events. Recall that, by construction, the Gaussian copula has zero tail dependence. The t–copula allows for dependence but imposes symmetry in dependence in the upper and lower tails of the distribution. The Clayton and rotated Gumbel copulas allow for lower tail dependence but impose zero upper tail dependence while the opposite is true for the rotated Clayton and the Gumbel copulas.

Selection among the alternative copula models can be guided through a consideration of the log likelihood function values, the AIC criteria, and the Cramér von Mises statistics. As we have noted, criteria based on likelihood functions may not be fully comparable across different parametric families. Thus, the Cramér von Mises statistic is our preferred model selection criterion. The rotated Clayton copula is supported in three of the six cases (Eastern Canada and the Northeast U.S., Eastern Canada and the Southeast U.S., and Eastern Canada and the Southwest U.S.). All three price pairs that involve Southwest U.S. markets favor Gaussian and t–copulas, though in the latter case, the high degrees of freedom for the t–copula estimates indicates a relationship very similar to that of the Gaussian, with no tail dependence. Estimates of the asymmetric Archimedean copulas (variants of the Clayton and Gumbel copulas) all indicate relatively strong tail dependence. Interpretation of tail dependence in cases where such dependence is only allowed in one tail can be aided by a consideration of the typical basis relationships among markets. In particular, to the extent that one market tends to export to another, we generally expect to see price differences tending to be either positive or negative, but not both. This reflects the presence of transactions costs which are a component of basis price differences. The asymmetry in commodity flows is a relatively common feature in most basic commodity markets, including manufactured wood products. Figure 2 presents nonparametric densities for the price differentials for all six pairs of markets. In five of six cases illustrated in Figure 1, definite patterns of basis, reflecting a relationship where one market price is generally above another, are indicated. Further, skewness is apparent in all of the nonparametric densities illustrated in Figure 2, indicating that large departures from price equality are more common in one direction than another. This suggests that the asymmetric tail dependence associated with the Clayton and Gumbel copulas (and their rotated versions) may be appropriate.

In four of the six cases considered, a greater degree of tail dependence is indicated by the preferred specification than would be implied by a standard Gaussian copula. The measure of dependence indicates that a standard Gaussian copula is inappropriate in modeling the price linkages. Of course, these measures of tail dependence are necessarily constrained by the parametric copula functions. In addition to examining tail dependence measures, an appropriate way to characterize the market integration relationships among the market pairs is to consider the joint *p.d.f.*'s implied by the copula estimates. To this end, we consider two alternative approaches to evaluating the joint distributions. In the first, we evaluate the joint *p.d.f.* implied by the copula for the actual sample data. This preserves the marginal distributions while, at the same time, permitting an evaluation of the patterns of correlation and dependence inherent in the sample. In a second approach, we evaluate the copula estimates using standard normal marginal distributions. Specifically, we evaluate the copula estimates across an evenly–spaced grid of random variables drawn from standard Gaussian marginals.[13] This allows a more direct illustration of the patterns of correlation and dependence inherent in the estimated copula function. Our choice of the quantile of the

[13] Although the sample of 812 observations allows accurate estimation of the copula parameters, it is relatively thin for the purposes of illustrating the joint distribution. Thus, we also utilize the much denser grid of values generated from a standard normal.

cumulative distribution of price differentials is motivated by this aspect of our analysis in that the quantiles are transparent to any specific marginal, reflecting their normalized nature. We simulate the joint distributions implied by the copula estimates that were favored by the Cramér von Mises statistics.

Contours of the resulting joint densities are presented for the favored copula functions in Figures 4 and 5. Specific relationships reflecting dependence or correlation among the lagged price differentials and differenced price differentials represent the distribution of the available data when the plots are evaluated for the sample data. In such cases, patterns of correlation or dependence are difficult to discern using only the sample data. When illustrated using Gaussian marginal distributions, the densities further illustrate patterns of tail dependence, where linkages between markets appear to be stronger for larger deviations from price parity. In particular, estimates of the rotated Clayton and Gumbel copulas in panels (b) and (d) of Figure 4 and panel (b) of Figure 5 illustrate tighter correlation in the tails, corresponding to stronger price adjustments when price differences are higher. That said, the patterns are subtle and may reflect the fact that each copula function is relatively restrictive in terms of the extent of tail dependence permitted.

We next consider allowing greater flexibility in the market relationships represented by the copula function estimates by allowing shape, correlation, and degrees of freedom parameters to vary according to the distribution of the price differentials. In particular, we allow parameters to vary with the empirical quantile of the lagged price differential. The intuition underlying such a specification is that the dependence among the first–differenced relative prices and the lagged values may be nonlinear, reflecting the influences of unobservable transactions costs. In particular, different patterns of adjustment may be implied for large deviations from spatial equilibrium, represented by large price differentials. The quantile of the distribution of price differences provides a normalized value that can be evaluated across different distributions, including the Gaussian marginals. Standard maximum likelihood estimation techniques are used to obtain parameter estimates.[14] The resulting

[14] We use a simulated annealing stochastic search algorithm to obtain starting values for standard quasi-Newton optimization procedures.

parameter estimates are presented in Table 3. It is important to note that the standard copula functions presented in Table 2 are nested within the specifications presented in Table 3. This allows standard likelihood ratio tests of the parameters and alternative specifications.

The state–dependent versions of the copula models provide substantial improvements in fit over the standard versions in several (but not all) cases. The Cramér von Mises statistics favor the state–dependent versions of the copula models in four of the six cases, with a fifth case amounting to a virtual tie. Likelihood function values favor the augmented t–copula in four of the six cases. The augmented Gumbel copula is favored in two cases by the AIC and the augmented rotated Clayton and Gumbel copulas receive support in two cases each. It is relevant to note that the parameter estimates corresponding to the state dependence effect (i.e., the coefficients on the empirical $c.d.f.$ values of the lagged price differences) are frequently statistically significant, even in cases where the copula is not favored over alternatives by the log–likelihood and AIC values. That said, the state–dependent model coefficients are not statistically significant in the majority of the single-parameter (Archimedean) copula models, including the rotated Clayton copulas that are frequently favored (Table 2).

In order to consider the distributional properties that underlie price linkages among the market pairs, we again simulate the implied joint $p.d.f.$ functions. In all but one case, we present examples for each pair that were given support by the model fitting criteria.[15] Figure 6 presents the resulting joint $p.d.f.$ contours. Subtle differences showing differing patterns of correlation and tail dependence are illustrated in the figures. In some cases, the correlation appears to be stronger for positive deviations while in others correlation appears much stronger for negative price differences, which reflects the basis patterns illustrated in Figure 2.

In five of the six cases, the coefficients corresponding to the time–varying effect for the degrees of freedom and correlation parameters in the t–copulas are statistically significant.

[15] In case of the market pair of the North Central U.S. and the Southwest U.S., we present simulations based on the rotated Gumbel, which has a statistically significant state-dependence parameter and an almost equivalent value of the Cramér von Mises statistic relative to the preferred model in Table 3.

Therefore, tail dependence may be variable across different values of the lagged price differentials, reflecting nonlinearities in the regional price relationships. Figure 7 illustrates the state–dependence effect on tail dependence implied by the t–copula models. Extreme values of the lagged price differentials, which represent departures from spatial equilibrium, appear to result in significantly different patterns of tail dependence in the t–copula models. For Eastern Canada and the North–Central US, the coefficients in the state-dependent copula models are not statistically significant. However, in the remainder of the market pairs, the coefficient estimates suggest a statistically significant effect of price differentials on tail dependence. These patterns are largely consistent with the basis relationships noted above in that tail dependence increases at either very small values of the price differential or very large values. In those markets that typically display a positive basis (see panels (b), (c), and (e) in Figure 2), tail dependence strengthens for large negative values of the price differential. The opposite effect is noted for markets with opposite basis patterns or in situations where no clear basis pattern exists.[16]

In accordance with existing research, the results indicate that market adjustments are generally larger in response to large price differences which reflect more substantial disequilibrium conditions (and therefore bigger arbitrage opportunities). The implications are very similar to those provided in other estimation approaches that allow for nonlinearities. In particular, regime switching and threshold models generally imply that price linkages and adjustment patterns are stronger and quicker when deviations from equilibrium are large. This suggests the presence of transactions costs and Heckscher's "commodity points."

[16]We also considered a more flexible copula model that would allow for asymmetric, non–zero tail dependence in each tail of the joint distribution. In particular, we adopt the Symmetric Joe-Clayton copula model that is introduced by Patton (2006). In most cases, we encountered significant problems obtaining convergent estimates of the copula parameters. This likely reflects the fact that data tend to be more concentrated in one or the other tail. Application of alternative copulas with greater flexibility remains a topic of current research.

4 Summary and Concluding Remarks

We evaluate the adherence to the economic conditions typically required for efficiently linked markets by considering the degree and nature of correlation implied by copula models of joint distributions of spatially related prices. To allow and model nonlinear behavior that might be caused by transactions costs, we adopt specific classes of copula functions that allow for "state–dependent" correlation and dependence, where the state is defined by the degree of market disequilibrium represented by spatial price differences at any point in time. We find that transactions costs bands are implied by certain nonlinear patterns of correlation. In addition, we consider more flexible copula models that allow parameters of the joint distributions to vary according to the "state" of market disequilibrium. We find that such models provide even stronger evidence of nonlinearities in market linkages.

One weakness of the copula approach is that it is usually difficult to select a specific parametric copula. We highlight alternative model fitting criteria that may be of value in comparing alternative copula models. Such an approach is, however, hindered by the fact that such comparisons do not necessarily comprise formal specification tests. Further, model fitting criteria may not be fully comparable across different copula families. An exception exists in the case of the Cramér-von Mises statistics, which we use in selecting our preferred model specifications. Additional research is needed on model selection criteria for nested and non–nested copula models.

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Table 1. OLS Estimates of Autoregressive Error-Correction Price Parity Model

$$\Delta(p_t^i - p_t^j) = a - b(p_{t-1}^j - p_{t-1}^i)$$

Parameter	Estimate	Standard Error	t-Ratio	Deviation Half-Life	R^2
.... Eastern Canada and NC US					
a	−0.0066	0.0013	−5.26		0.0555
b	0.1136	0.0165	6.89	5.75	
.... Eastern Canada and SE US					
a	−0.0019	0.0012	−1.60		0.0217
b	0.0467	0.0110	4.24	14.51	
.... Eastern Canada and SW US					
a	−0.0045	0.0013	−3.40		0.0391
b	0.0815	0.0142	5.74	8.15	
.... NC US and SE US					
a	0.0008	0.0010	0.82		0.0264
b	0.0534	0.0114	4.68	12.62	
.... NC US and SW US					
a	0.0002	0.0010	0.23		0.0576
b	0.1148	0.0163	7.03	5.68	
.... SE US and SW US					
a	−0.0007	0.0007	−0.94		0.0279
b	0.0563	0.0117	4.81	11.97	

An asterisk indicates statistical significance at the $\alpha = .10$ or smaller level. Deviation half-lives represent the weeks required to eliminate one-half of the deviation from equilibrium and are given by $ln(0.5)/ln(1-\beta)$.

Table 2. Copula Parameter Estimates (With Empirical Marginals)

$$C(\Delta(p_t^i - p_t^j), (p_{t-1}^i - p_{t-1}^j))$$

Copula	Parameter	Estimate	Standard Error	t Ratio	Log Likelihood	AIC	Cramér von Mises	Lower Tail Dependence	Upper Tail Dependence
Eastern Canada and NC US									
Gaussian	ρ	0.2321	0.0337	6.90*	21.8316	−41.6632	0.0027	0.0000	0.0000
T	ρ	0.2265	0.0337	6.72*	22.4752	−40.9503	0.0027	0.0007	0.0007
	ν	22.7499	5.1802	4.39*					
Clayton	θ	0.2330	0.0494	4.72*	14.9767	−27.9534	0.0050	0.0511	0.0000
Rotated Clayton	θ	0.2724	0.0486	5.61*	19.9210	−37.8420	0.0021†	0.0000	0.0785
Gumbel	θ	1.1504	0.0281	40.96*	21.1173	−40.2345	0.0021	0.0000	0.1733
Rotated Gumbel	θ	1.1391	0.0281	40.47*	18.9214	−35.8428	0.0044	0.1623	0.0000
Eastern Canada and SE US									
Gaussian	ρ	0.1530	0.0345	4.43*	9.3252	−16.6505	0.0028	0.0000	0.0000
T	ρ	0.1510	0.0346	4.37*	9.4024	−14.8049	0.0028	0.0000	0.0000
	ν	96.9999	1.0641	91.16*					
Clayton	θ	0.1328	0.0455	2.92*	5.6982	−9.3964	0.0039	0.0054	0.0000
Rotated Clayton	θ	0.1742	0.0408	4.27*	9.4390	−16.8780	0.0023†	0.0000	0.0187
Gumbel	θ	1.0859	0.0237	45.78*	8.0347	−14.0693	0.0024	0.0000	0.1067
Rotated Gumbel	θ	1.0774	0.0241	44.70*	7.7772	−13.5544	0.0034	0.0972	0.0000
Eastern Canada and SW US									
Gaussian	ρ	0.1968	0.0340	5.79*	15.5670	−29.1340	0.0022	0.0000	0.0000
T	ρ	0.1949	0.0342	5.70*	15.7462	−27.4924	0.0021†	0.0000	0.0000
	ν	46.9979	23.8265	1.97*					
Clayton	θ	0.2086	0.0454	4.59*	12.4103	−22.8206	0.0032	0.0360	0.0000
Rotated Clayton	θ	0.2034	0.0462	4.40*	11.6107	−21.2214	0.0032	0.0000	0.0331
Gumbel	θ	1.1177	0.0270	41.42*	12.5964	−23.1927	0.0026	0.0000	0.1408
Rotated Gumbel	θ	1.1195	0.0266	42.07*	13.2852	−24.5703	0.0027	0.1426	0.0000

An asterisk indicates statistical significance at the $\alpha = .10$ or smaller level. Minimum values of the Cramér von Mises statistic are identified by "†". Note that likelihood function values and AIC statistics may not be fully comparable across different copula families and are presented for comparison purposes only.

Table 2. (continued)

Copula	Parameter	Estimate	Standard Error	t Ratio	Log Likelihood	AIC	Cramér von Mises	Lower Tail Dependence	Upper Tail Dependence
NC US and SE US									
Gaussian	ρ	0.1859	0.0347	5.36*	13.8513	−25.7027	0.0068	0.0000	0.0000
T	ρ	0.1807	0.0348	5.18*	14.4285	−24.8569	0.0069	0.0004	0.0004
	ν	23.3336	0.5798	40.25*					
Clayton	θ	0.0940	0.0462	2.03*	2.6494	−3.2988	0.0093	0.0000	0.0006
Rotated Clayton	θ	0.2949	0.0478	6.17*	24.2816	−46.5631	0.0038†	0.0953	0.0000
Gumbel	θ	1.1403	0.0254	44.85*	21.1548	−40.3096	0.0049	0.0000	0.1635
Rotated Gumbel	θ	1.0753	0.0272	39.57*	5.9797	−9.9594	0.0084	0.0948	0.0000
NC US and SW US									
Gaussian	ρ	0.2436	0.0346	7.04*	24.1257	−46.2515	0.0022†	0.0000	0.0000
T	ρ	0.2360	0.0358	6.59*	27.1568	−50.3136	0.0025	0.0321	0.0321
	ν	9.0134	3.8569	2.34*					
Clayton	θ	0.2323	0.0487	4.77*	14.1956	−26.3911	0.0061	0.0506	0.0000
Rotated Clayton	θ	0.3210	0.0540	5.94*	26.4397	−50.8794	0.0035	0.0000	0.1154
Gumbel	θ	1.1792	0.0299	39.50*	29.3536	−56.7073	0.0025	0.0000	0.2000
Rotated Gumbel	θ	1.1504	0.0304	37.82*	18.2574	−34.5148	0.0042	0.1733	0.0000
SE US and SW US									
Gaussian	ρ	0.2052	0.0357	5.75*	16.9584	−31.9168	0.0144	0.0000	0.0000
T	ρ	0.2027	0.0365	5.55*	19.5611	−35.1221	0.0143†	0.0192	0.0192
	ν	10.2438	4.9235	2.08*					
Clayton	θ	0.2543	0.0529	4.81*	17.1453	−32.2907	0.0169	0.0655	0.0000
Rotated Clayton	θ	0.2000	0.0452	4.43*	11.0723	−20.1446	0.0212	0.0000	0.0313
Gumbel	θ	1.1226	0.0287	39.14*	12.6694	−23.3388	0.0187	0.0000	0.1459
Rotated Gumbel	θ	1.1453	0.0287	39.97*	19.3011	−36.6022†	0.0152	0.1683	0.0000

An asterisk indicates statistical significance at the $\alpha = .10$ or smaller level. Minimum values of the Cramér von Mises statistic are identified by "†". Note that likelihood function values and AIC statistics may not be fully comparable across different copula families and are presented for comparison purposes only.

Table 3. State-Varying Copula Parameter Estimates (With Empirical Marginals)

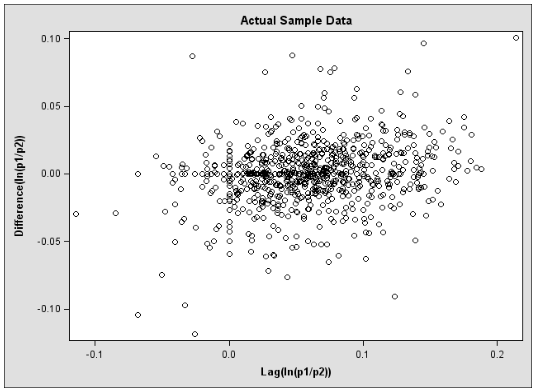
Copula	Parameter	Estimate	Standard Error	t Ratio	Log Likelihood	AIC	Cramér von Mises
Eastern Canada and NC US							
Gaussian	α_0	0.4722	0.1160	4.07*	21.8477	−39.6955	0.0027
	α_1	−0.0306	0.1752	−0.17			
T	β_0	3.1118	0.2334	13.33*	22.4884	−36.9769	0.0028
	β_1	−0.0627	0.1775	−0.35			
	α_0	0.4598	0.1155	3.98*			
	α_1	−0.0294	0.1771	−0.17			
Clayton	γ_0	−1.6088	0.3156	−5.10*	15.2875	−26.5751	0.0047
	γ_1	0.4299	0.5653	0.76			
Rotated Clayton	γ_0	−1.5461	0.2948	−5.25*	20.8664	−37.7327	0.0018†
	γ_1	0.6750	0.5094	1.33			
Gumbel	γ_0	−1.4855	0.2844	−5.22*	22.2578	−40.5155	0.0023
	γ_1	−0.7259	0.4846	−1.50			
Rotated Gumbel	γ_0	−1.6248	0.3179	−5.11*	19.6744	−35.3487	0.0055
	γ_1	−0.6186	0.5182	−1.19			
Eastern Canada and SE US							
Gaussian	α_0	0.1614	0.1213	1.33	10.4785	−16.9569	0.0024
	α_1	0.2605	0.1724	1.51			
T	β_0	1.7962	0.0046	393.17*	12.2880	−16.5759	0.0025
	β_1	5.9694	0.0279	213.92*			
	α_0	0.1136	0.1255	0.91			
	α_1	0.3149	0.1759	1.79*			
Clayton	γ_0	−3.0667	2.2884	−1.34	7.9397	−11.8794	0.0028
	γ_1	2.1981	2.8932	0.76			
Rotated Clayton	γ_0	−1.7492	0.2849	−6.14*	9.4391	−14.8781	0.0022†
	γ_1	0.0051	0.5748	0.01			
Gumbel	γ_0	−2.3758	0.5101	−4.66*	8.0505	−12.1010	0.0023
	γ_1	−0.1272	0.6718	−0.19			
Rotated Gumbel	γ_0	−1.6544	0.3429	−4.83*	10.0751	−16.1502	0.0060
	γ_1	−1.6406	0.9143	−1.79*			
Eastern Canada and SW US							
Gaussian	α_0	0.3817	0.1142	3.34*	15.5670	−27.1340	0.0021
	α_1	0.0005	0.1675	0.00			
T	β_0	2.8031	8.7312	0.32	16.0060	−24.0119	0.0021†
	β_1	1.7504	0.8010	2.19*			
	α_0	0.3816	0.1248	3.06*			
	α_1	−0.0052	0.1695	−0.03			
Clayton	γ_0	−1.7806	0.3317	−5.37*	12.9265	−21.8530	0.0030
	γ_1	0.5979	0.5495	1.09			
Rotated Clayton	γ_0	−1.8161	0.3244	−5.60*	12.2014	−20.4028	0.0030
	γ_1	0.6317	0.5720	1.10			
Gumbel	γ_0	−1.6868	0.3298	−5.11*	13.6599	−23.3198	0.0021
	γ_1	−0.8052	0.5349	−1.51			
Rotated Gumbel	γ_0	−1.7798	0.3265	−5.45*	13.8704	−23.7409	0.0030
	γ_1	−0.6024	0.5319	−1.13			

An asterisk indicates statistical significance at the $\alpha = .10$ or smaller level. Minimum values of the Cramér von Mises statistic are identified by "†". Note that likelihood function values and AIC statistics may not be fully comparable across different copula families and are presented for comparison purposes only.

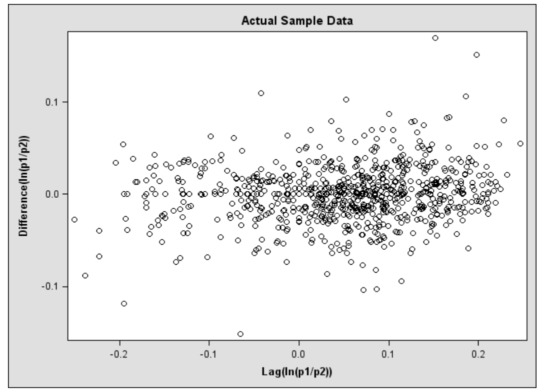
Table 3. (continued)

Copula	Parameter	Estimate	Standard Error	t Ratio	Log Likelihood	AIC	Cramér von Mises
NC US and SE US							
Gaussian	α_0	0.1235	0.1144	1.08	17.2462	−30.4924	0.0050
	α_1	0.4496	0.1767	2.54*			
T	β_0	4.9419	0.7371	6.70*	18.6964	−29.3928	0.0048
	β_1	−2.9548	0.7400	−3.99*			
	α_0	0.1194	0.1144	1.04			
	α_1	0.4479	0.1795	2.50*			
Clayton	γ_0	−15.4014	6.0202	−2.56*	12.1436	−20.2873	0.0067
	γ_1	15.8854	6.3794	2.49*			
Rotated Clayton	γ_0	−1.0931	0.2066	−5.29*	24.6527	−45.3055	0.0040†
	γ_1	−0.4065	0.4634	−0.88			
Gumbel	γ_0	−2.4396	0.4616	−5.29*	22.0515	−40.1030	0.0044
	γ_1	0.7301	0.5664	1.29			
Rotated Gumbel	γ_0	−0.4673	0.3334	−1.40	16.8480	−29.6960	0.0168
	γ_1	−12.0656	4.8408	−2.49*			
NC US and SW US							
Gaussian	α_0	0.3899	0.1137	3.43*	24.7267	−45.4534	0.0021†
	α_1	0.1814	0.1710	1.06			
T	β_0	2.4357	0.5523	4.41*	27.6791	−47.3581	0.0023
	β_1	−0.6625	0.6551	−1.01			
	α_0	0.3910	0.1174	3.33*			
	α_1	0.1411	0.1816	0.78			
Clayton	γ_0	−1.8396	0.3520	−5.23*	15.9671	−27.9342	0.0053
	γ_1	0.9918	0.5320	1.86*			
Rotated Clayton	γ_0	−1.1999	0.2164	−5.54*	26.5556	−49.1113	0.0036
	γ_1	0.1915	0.4032	0.48			
Gumbel	γ_0	−1.6045	0.2810	−5.71*	29.4723	−54.9447	0.0023
	γ_1	−0.1922	0.3840	−0.50			
Rotated Gumbel	γ_0	−1.3222	0.2614	−5.06*	20.9626	−37.9253	0.0053
	γ_1	−1.0827	0.4962	−2.18*			
SE US and SW US							
Gaussian	α_0	0.5267	0.1201	4.39*	18.0378	−32.0756	0.0155
	α_1	−0.2445	0.1719	−1.42			
T	β_0	1.6808	0.5253	3.20*	21.1449	−34.2898	0.0149†
	β_1	1.1726	0.5989	1.96*			
	α_0	0.5129	0.1268	4.04*			
	α_1	−0.2336	0.1792	−1.30			
Clayton	γ_0	−1.2689	0.2319	−5.47*	17.3373	−30.6746	0.0176
	γ_1	−0.3083	0.5377	−0.57			
Rotated Clayton	γ_0	−2.0613	0.3757	−5.49*	13.2789	−22.5577	0.0183
	γ_1	1.1977	0.5587	2.14*			
Gumbel	γ_0	−1.3420	0.2777	−4.83*	16.3131	−28.6261	0.0166
	γ_1	−1.4288	0.5729	−2.49*			
Rotated Gumbel	γ_0	−2.0178	0.3677	−5.49*	19.3503	−34.7007	0.0155
	γ_1	0.1479	0.4620	0.32			

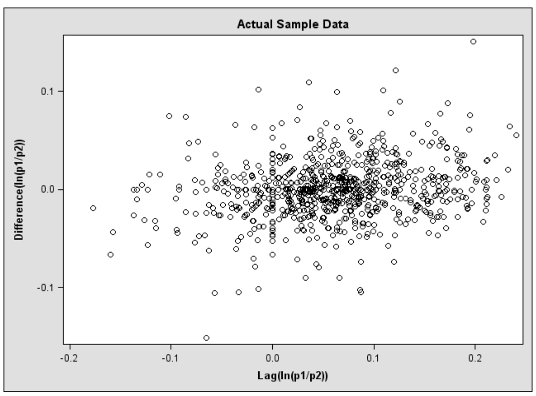
An asterisk indicates statistical significance at the $\alpha = .10$ or smaller level. Minimum values of the Cramér von Mises statistic are identified by "†". Note that likelihood function values and AIC statistics may not be fully comparable across different copula families and are presented for comparison purposes only.



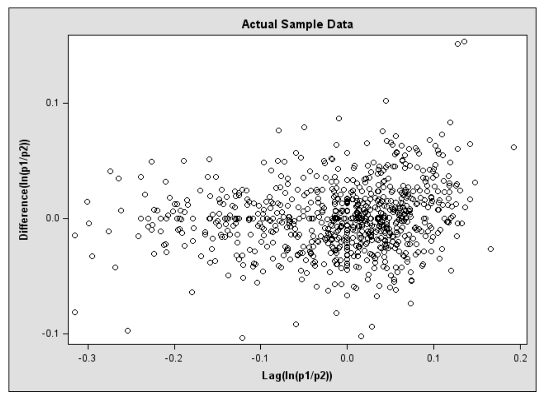
(a) Eastern Canada and NC US



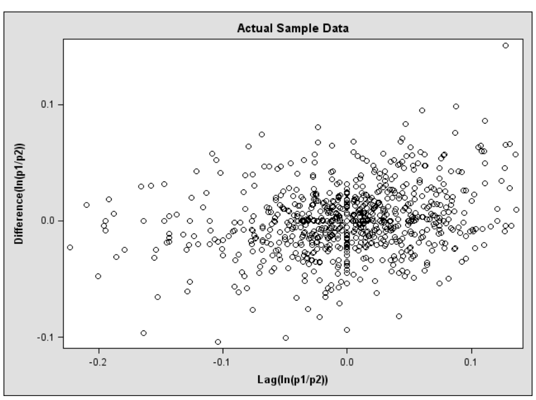
(b) Eastern Canada and SE US



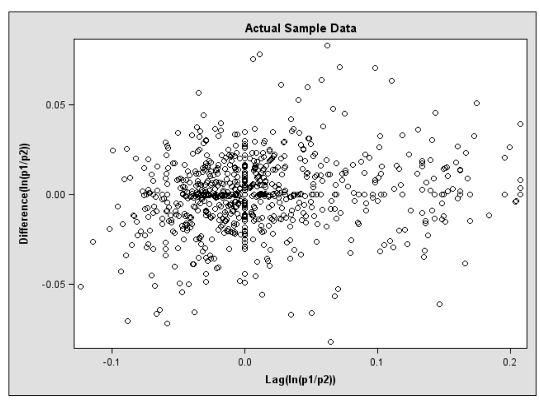
(c) Eastern Canada and SW US



(d) NC US and SE US

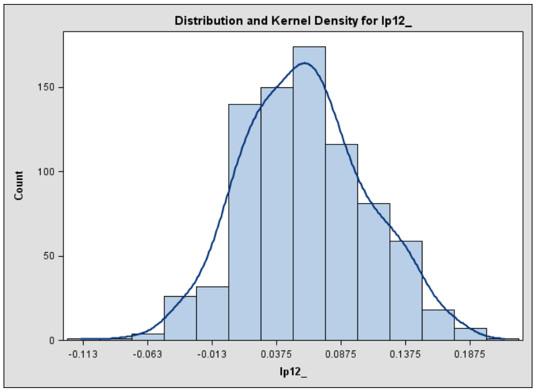


(e) NC US and SW US

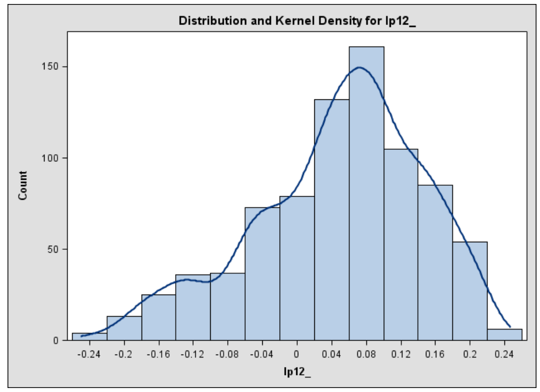


(f) SE US and SW US

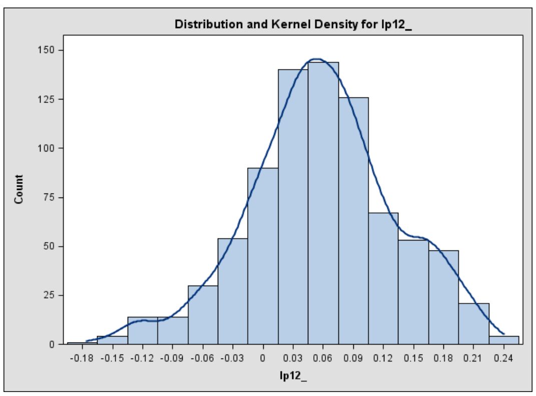
Figure 1: Regional OSB Prices Sample Data $(\Delta(p_t^i - p_t^j), (p_{t-1}^i - p_{t-1}^j))$



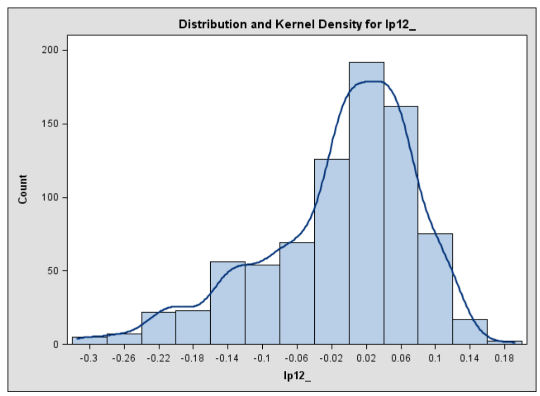
(a) Eastern Canada and NC US



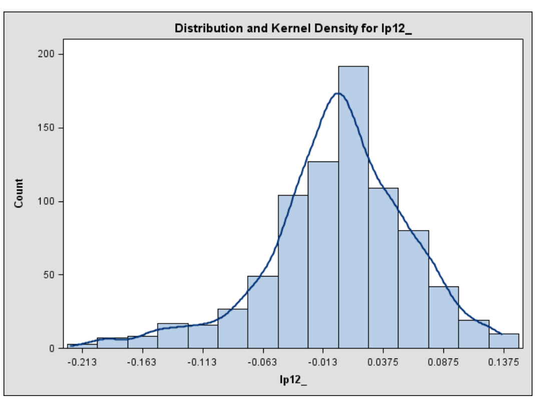
(b) Eastern Canada and SE US



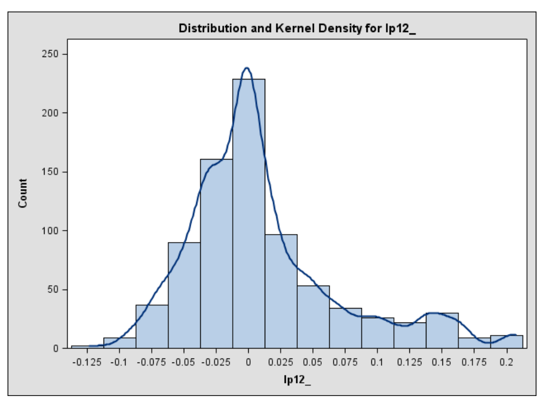
(c) Eastern Canada and SW US



(d) NC US and SE US

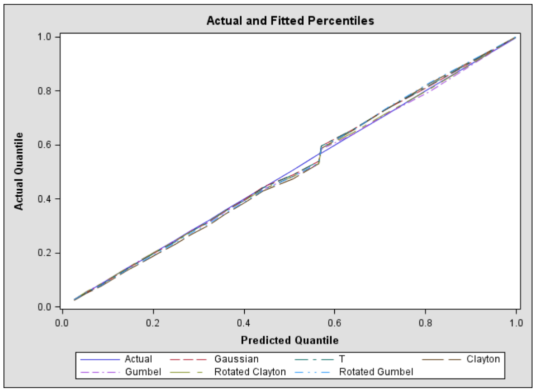


(e) NC US and SW US

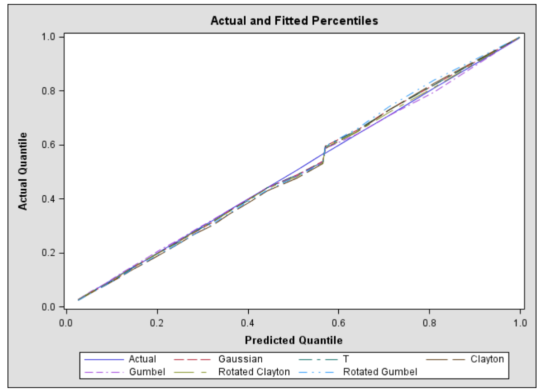


(f) SE US and SW US

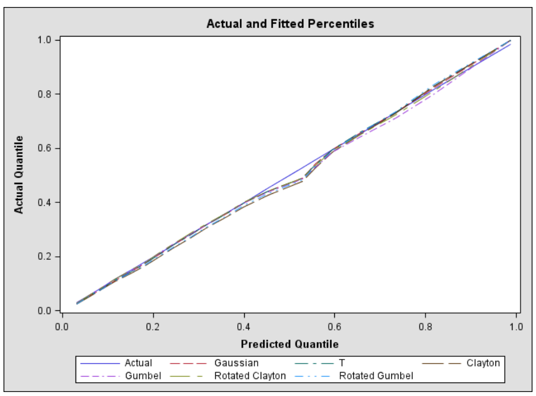
Figure 2: Distribution of Lagged Price Differentials $(p^i_{t-1} - p^j_{t-1})$



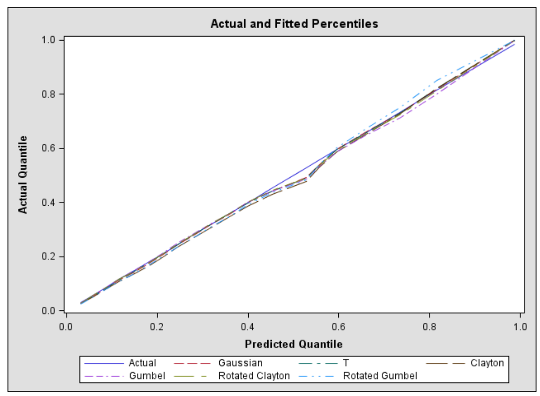
(a) Eastern Canada and NC US



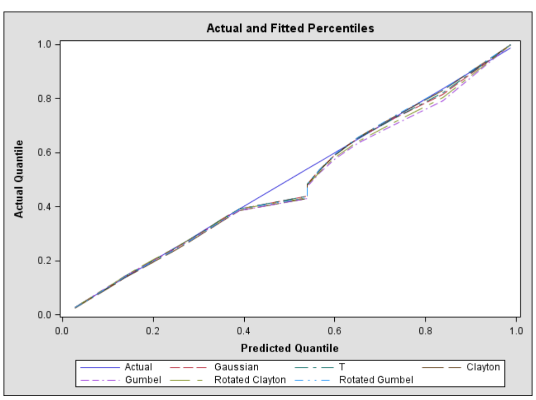
(b) Eastern Canada and NC US: State Dependent



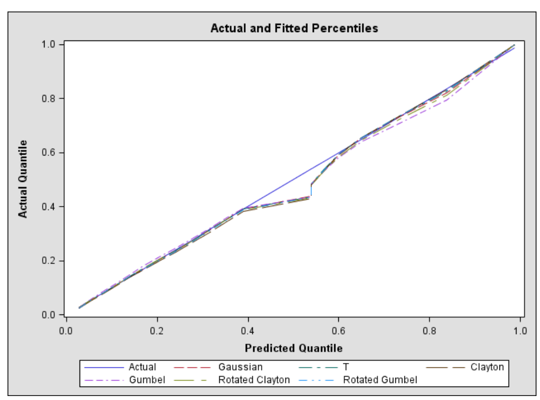
(c) NC US and SW US



(d) NC US and SW US: State Dependent

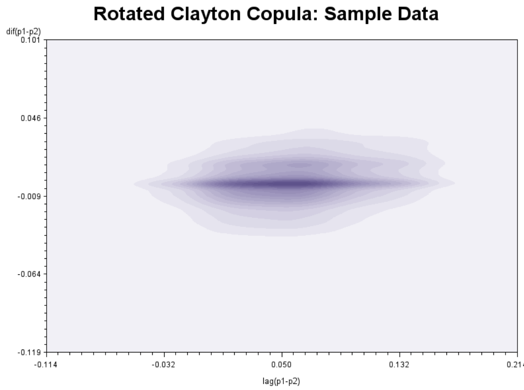


(e) SE US and SW US

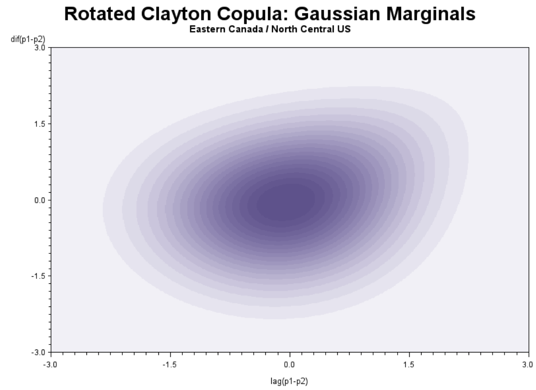


(f) SE US and SW US: State Dependent

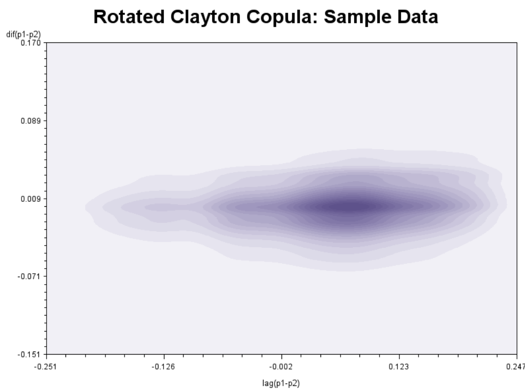
Figure 3: Actual and Fitted Quantiles from Simulated Copula Densities: Calculated at 0.05 Incremental Percentiles



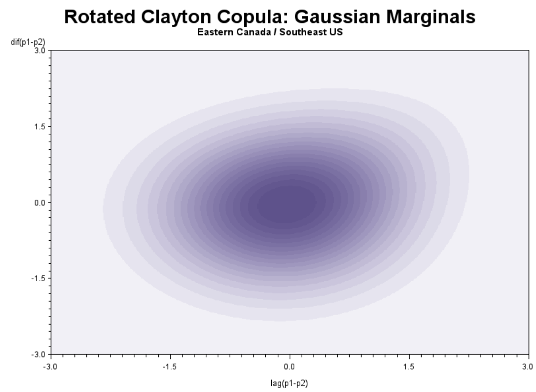
(a) Eastern Canada and NC US



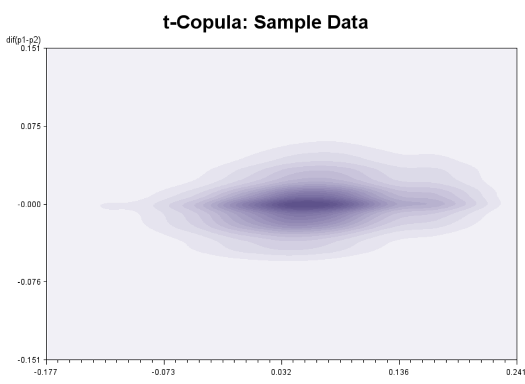
(b) Eastern Canada and NC US



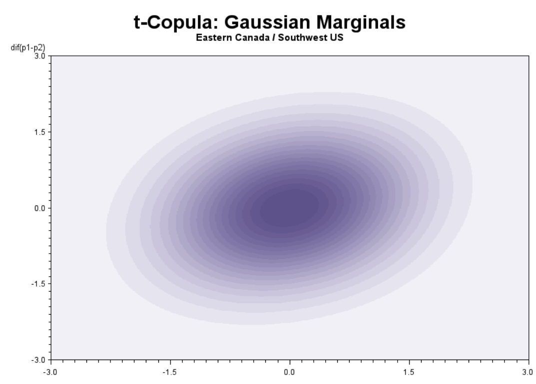
(c) Eastern Canada and SE US



(d) Eastern Canada and SE US

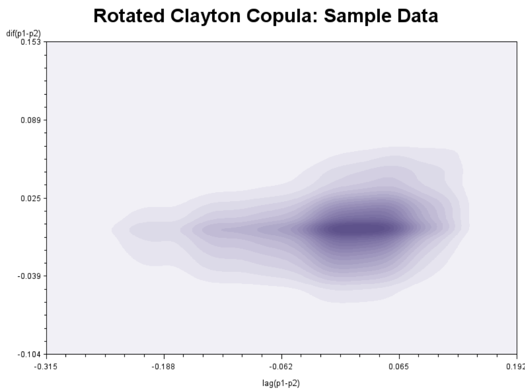


(e) Eastern Canada and SW US

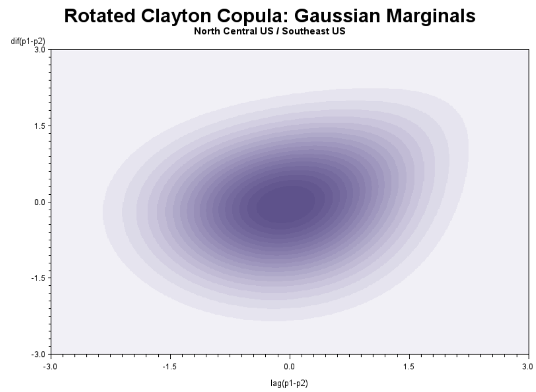


(f) Eastern Canada and SW US

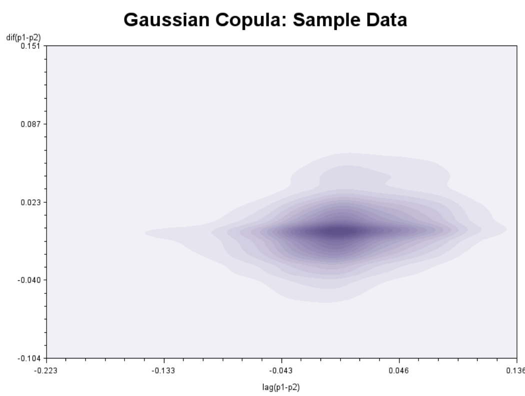
Figure 4: Contours of Estimated Copula Joint Probability Functions (Using Sample Data and with Standard Normal Marginals)



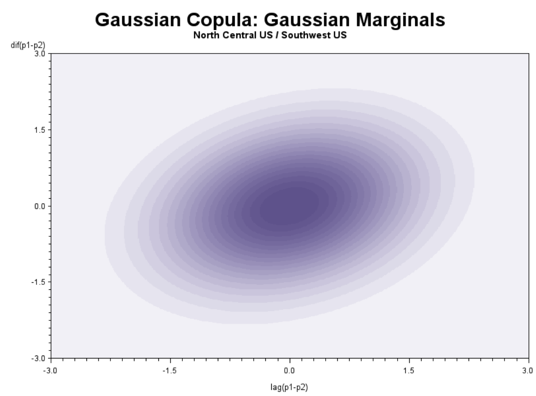
(a) NC US and SE US



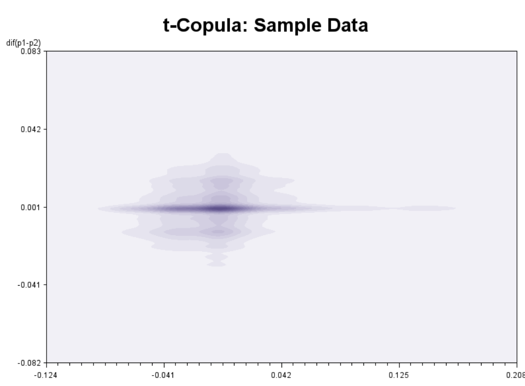
(b) NC US and SE US



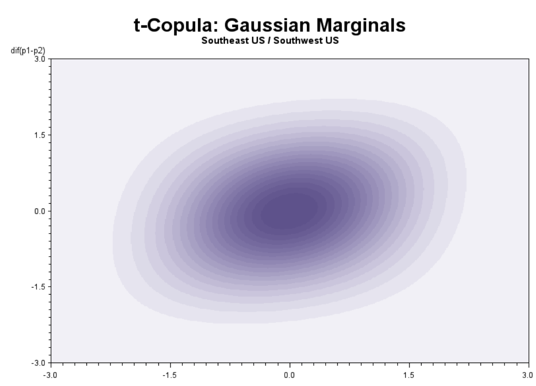
(c) NC US and SW US



(d) NC US and SW US

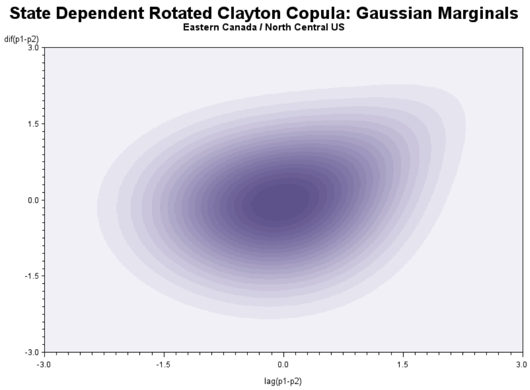


(e) SE US and SW US

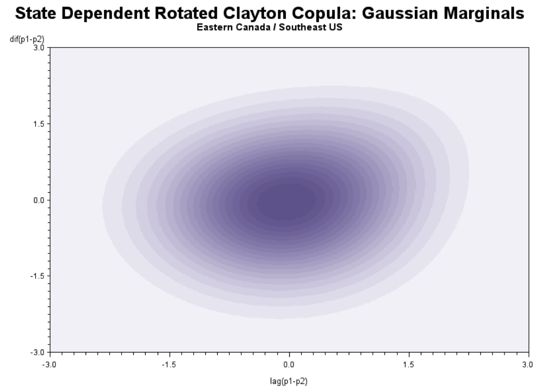


(f) SE US and SW US

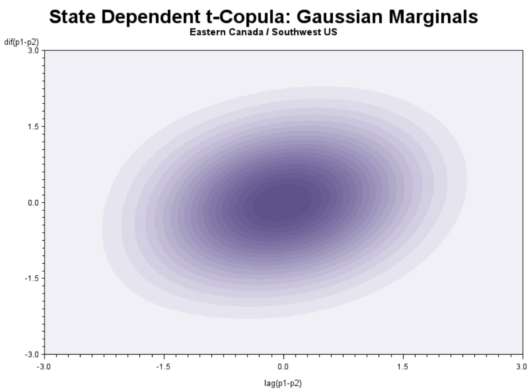
Figure 5: Contours of Estimated Copula Joint Probability Functions (Using Sample Data and with Standard Normal Marginals)



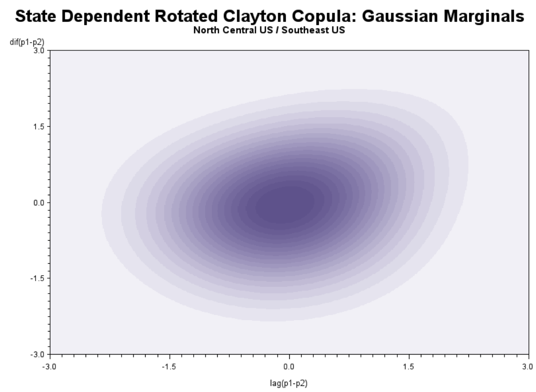
(a) Eastern Canada and NC US



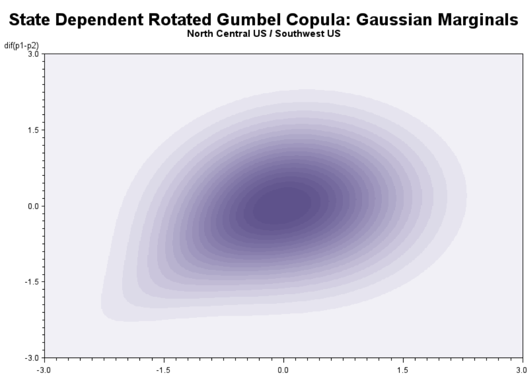
(b) Eastern Canada and SE US



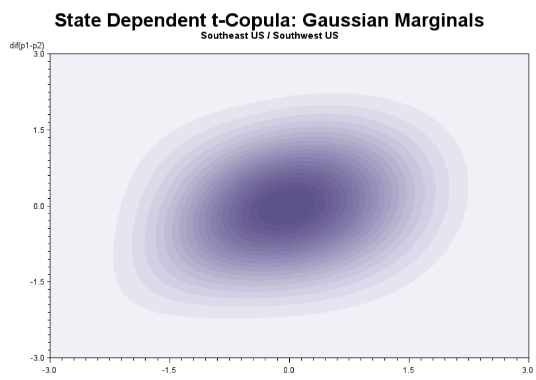
(c) Eastern Canada and SW US



(d) NC US and SE US

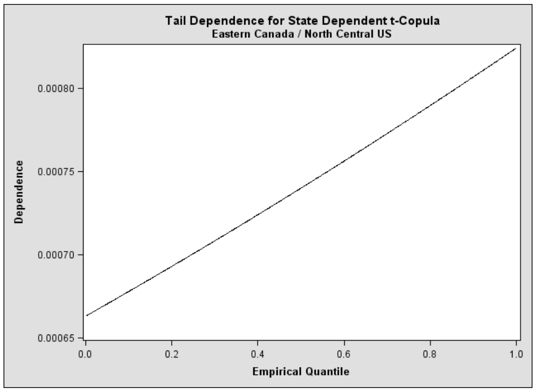


(e) NC US and SW US

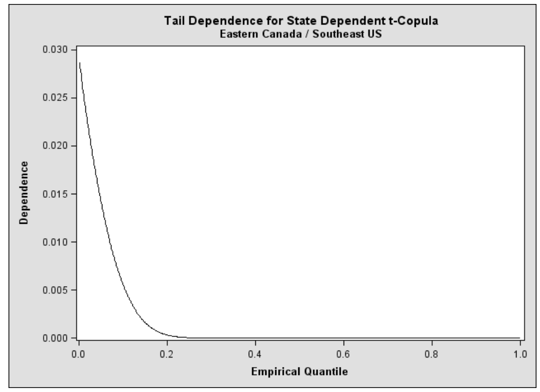


(f) SE US and SW US

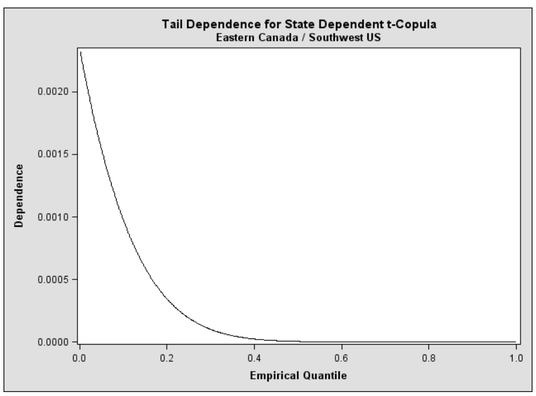
Figure 6: Contours of Estimated State-Dependent Copula Joint Probability Functions (With Standard Normal Marginals)



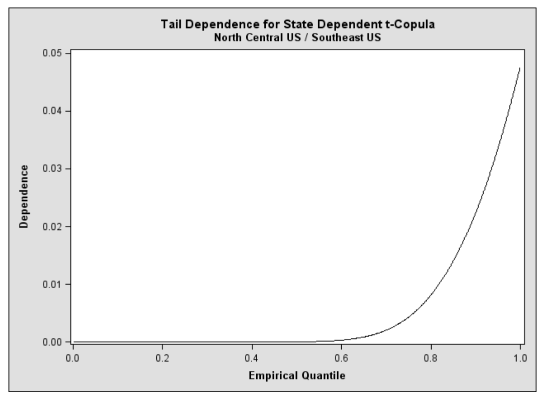
(a) Eastern Canada and NC US



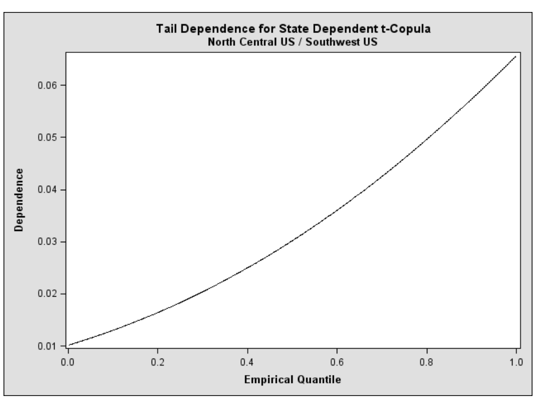
(b) Eastern Canada and SE US



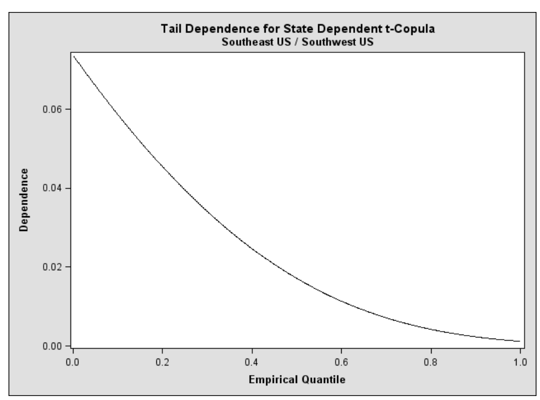
(c) Eastern Canada and SW US



(d) NC US and SE US



(e) NC US and SW US



(f) SE US and SW US

Figure 7: Variable Tail Dependence in T-Copulas as a Function of Quantiles of Lagged Price Differentials)