Bob, Alice and Carol are playing frisbee. Bob always throws to Alice and Alice always throws to Carol. Carol throws to Bob \(\frac{1}{3}\) of the time and to Alice \(\frac{2}{3}\) of the time.

a) Write the Transition matrix for the situation:

\[ T = \begin{pmatrix}
A & B & C \\
0 & 0 & 1 \\
1 & 0 & 0 \\
\frac{2}{3} & \frac{1}{3} & 0
\end{pmatrix} \]

b) If Carol has the frisbee now, what's the probability she has it after it has been thrown twice?

\[ T^2 = \begin{pmatrix}
A & B & C \\
\frac{2}{3} & \frac{1}{3} & 0 \\
0 & 1 & 0 \\
\frac{1}{3} & 0 & \frac{2}{3}
\end{pmatrix} \]

\[ P_{33}^{(2)} = \frac{2}{3} \]

C) If the probability that Alice, Bob and Carol have the frisbee initially are \(0.1, 0.5, 0.4\) respectively, what is the probability Carol has the frisbee after 2 throws?

\[ P_0 = (0.1, 0.5, 0.4) \]

\[ P_2 = P_0 T^2 = (0.1, 0.5, 0.4) \begin{pmatrix}
\frac{2}{3} & \frac{1}{3} & 0 \\
0 & 1 & 0 \\
\frac{1}{3} & 0 & \frac{2}{3}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{5} & \frac{1}{30} & \frac{23}{30}
\end{pmatrix} \]

Prob Carol has it after 2 throws is \(\frac{23}{30}\)