

10pts

9) Bob, Alice and Carol are playing frisbee. Bob always throws to Alice and Alice always throws to Carol. Carol throws to Bob  $\frac{2}{3}$  of the time and to Alice  $\frac{1}{3}$  of the time.

a) Write the Transition matrix for the situation.

4pt

$$T = \begin{matrix} & \begin{matrix} Alice & Bob & Carol \end{matrix} \\ \begin{matrix} Alice \\ Bob \\ Carol \end{matrix} & \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ \frac{1}{3} & \frac{2}{3} & 0 \end{pmatrix} \end{matrix}$$

b). If Carol has the frisbee now, what's the probability she has it after it has been thrown twice?

2pt for T<sup>2</sup>

$$T^2 = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 \\ \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix} \end{matrix}$$

$$P_{33}^{(2)} = \frac{1}{3}$$

2pt Ans.

c) If the probability that Alice, Bob and Carol have the frisbee initially are .2, .4, .4 respectively, what is the probability Carol has the frisbee after 2 throws?

2pt

$$P_0 = (.2, .4, .4)$$

$$P_2 = P_0 T^2 = (.2, .4, .4) \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 \\ \frac{2}{3} & 0 & \frac{1}{3} \end{pmatrix} = \left( \frac{1}{3}, \frac{2}{15}, \frac{8}{15} \right)$$

Prob. Carol has it =  $\frac{8}{15}$